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ABSTRACT

The papers in this monograph are grouped by the five session topics of the Using Research Group meetings. Perspectives on Using Research was first, with papers on research and the teaching job (Romberg), utility (Wheeler), review and dissemination (Dessart), preparing materials (Jurdak), and the teacher's view (Williams). The second session was on Means of Dissemination of Research Results, with papers on that title (Clarkson), communicating research results (Firth; Akers and Silver), developing teachers' styles (Swan), and who interprets research (Clegg). Effects of Research on Teachers formed the third category, considering Logo (Kieren), teacher education (Comiti), beginning teachers (Blane), history (Abe), and concept analysis (Bergeron and Herscovics). In the fourth session, Teachers as Researchers, papers concerned teachers' role (Cooney), carryover from college to school (Stewart), rational numbers (Owens), student teachers (Mitchelmore), and implications of the International Study (Gilmer). The final session was on Effects of Research on School Practice, with papers on cognitive psychology (Becker), problem solving (Nagasaki and Hashimoto; DeCorte and others), novel activities (Hershkovitz and Zehavi), and mathematical games (Bright and Harvey). An epilog, with open discussion, followed. (MNS)

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ICME5

Fifth International Congress on Mathematical Education

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Using Research in the Professional Life of Mathematics Teachers

Edited by Thomas A. Romberg

Using Research Group:
Professional Life of Teachers Theme, ICME5

Organized by Donald J. Dessart
and Thomas A. Romberg

May 1985

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Using Research in the Professional Life of Mathematics Teachers

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Using Research Group:

Professional Life of Teachers Theme,

ICME5

Organizers: Donald J. Dessart
University of Tennessee

and

Thomas A. Romberg
University of Wisconsin-Madison

Wisconsin Center for Education Research
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PROLOGUE

The papers in this monograph were originally prepared for the meetings of the USING RESEARCH GROUP for the Fifth International Congress on Mathematical Education, Adelaide, Australia, August 24-30, 1984. The Group was one of five groups organized on the theme "The Professional Life of Teachers".

The four-day meetings of the "Using Research Group" were organized by Donald J. Dessart, University of Tennessee (USA) and Thomas A. Romberg, University of Wisconsin (USA). The session began with a panel discussion on "Perspectives on Using Research." This was followed by four paper presentation sessions, one discussing "Means of Dissemination of Research;" a second covering "Effects of Research on School Practice." In all of these sessions, short paper presentations were followed by time for group discussion. A final session followed where participants assembled for an open discussion on "Future Directions."

The papers in this monograph are organized around those topics. The epilog is a summary of the discussion which took place at the final session. Following the meeting, the participants were given the opportunity to revise and edit their papers before publication.

The organizers wish to thank all the authors for their efforts in preparing, presenting, and revising their papers. Also, the organizers wish to thak Carolyn Kieren (Canada), Charleen DeRidder (USA), Laurie Hart Reyes (USA), and Brian Donovan (Australia) for chairing sessions at the meeting.

Finally, we wish to thank Chris Kruger for the final preparation of this monograph.

PERSPECTIVES ON USING RESEARCH

RESEARCH AND THE JOB OF TEACHING

Thomas A. Romberg, University of Wisconsin
USA

Research seems to have relatively little influence on the day-to-day work of teachers. It is hard to imagine a teacher who would refuse to teach students because he/she lacked research based knowledge about how students learn or about instruction. Lacking such knowledge would not phase most teachers, and schools would continue to operate pretty much as they do now. Furthermore, if teachers needed information to solve a problem, it is unlikely that they would search the research literature or ask a researcher to find an answer. In fact, as Bishop (1982) has pointed out, teachers, when faced with a problem, are most likely to seek "advice" from experienced teachers.

WHY IS IT THE CASE THAT TEACHERS DO NOT RELY ON RESEARCH?

Our intent in organizing the "Using Research Group" within the theme Professional Life of Teachers at ICME5 was to explore this question. In this brief introductory paper, I want to provide a starting point for the discussions by examining three of the many possible answers to this question:

1) Teachers are not professionals.
2) What is called "educational research" is not related to teaching.
3) The potential for a strong relationship between research and teaching has not been adequately developed.

PROFESSIONALISM AND THE JOB OF TEACHING

Teachers are considered as professional because they are highly trained and the demands of the job require judgment and decision making based on that training. These are the characteristics considered to be important in professions. But in education, does that training involve learning about research?; how it is carried out and how to interpret findings?; and do the actual judgments and decisions involved in teaching require such knowledge?

To illustrate the importance of these questions, let me point out that a surgeon could not perform open-heart surgery if he lacked research-based knowledge about heart functions, anesthesia, the meanings of symptoms, and the likely risks of certain actions. Without such knowledge derived from research, doctors would have no idea how to treat anything other than common ailments. Doctors are trained so that they understand research, so that they can use research based knowledge to make decisions in their practice, and if they have a problem, they are also trained to search the literature and to call on specialists for help. This illustration, while somewhat unfair to teachers because of differing circumstances, shows the power and potential of research-based knowledge on practice.

Does the job of teaching really involve judgments and decisions which could be based on research-based knowledge? In 1975 in the U.S., the National Advisory Committee on Mathematical Education (NACOME) commissioned a study of elementary school mathematics instruction. The picture drawn from that survey is as follows: The median classroom is self-contained. The mathematics period is about 43 minutes long, and about half of this time is spent on written work. A single text is used in whole-class instruction. The text is followed fairly closely, but students are likely to read at most one or two pages out of five pages of textual materials other than problems. For students, the text is primarily a source of problem lists (Conference Board of Mathematical Sciences, 1975, p. 77). Within the context, other studies commissioned by the National Science Foundation have shown that the daily sequence of activities involved in teaching mathematics involved:

> First, answers were given for the previous day's assignment. The more difficult pro9blems were worked at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to students working independently on the homework while the teacher moved about the room answering questions. The most noticeable thing about math classes was the repetition of this routine (Welch, 1978, p. 6).

From this picture of the typical classroom and the job teachers actually perform, it is hard to argue that teaching is really a profession. The teacher's job is related neither to a conception of mathematical knowledge to be transmitted, nor to an understanding of how learning occurs, nor to knowing the likely outcomes of various instructional actions. Elsewhere, I have argued that the job of teaching in the traditional classroom is managerial or procedural in that the "job is to assign lessons to their class of students, start and stop lessons according to some schedule, explain the rules and procedures of each lesson, judge the actions of students during the lesson, and maintain order and control throughout" (Romberg, 1985, p. 5). Thus, research on learning and teaching has little relevance because the judgments and decisions being made are not about learning, but about management.

In such situations, the teaching of mathematics is too often done without care or reflection. The job of teaching is perceived to be procedural or managerial and not adaptive. Too many teachers feel obligated to cover the book. Too few teachers see that student learning of mathematical methods and their use in solving problems is the primary goal of instruction. In the U.S. at the elementary school level, most teachers have an inadequate mathematical background. Now growing numbers of teachers at the secondary school level also are underprepared. To meet current shortages, many teachers are now being licensed with minimal preparation. This problem can only get worse during the next decade if the current trends in teacher education continue. Furthermore, teachers tend to be isolated in their own classrooms. They have little opportunity to share information with other staff members and little access to new knowledge (Tye & Tye, 1984).

The above picture may be real for many classrooms (at least in the USA) as they now operate, but it need not be the picture of classroom teaching as it should be. Currently, the job of teaching is carried on under impossible conditions. The most important feature of schools is that schooling is a collective experience. For the student, being in school means being in a crowd. For the teacher, it always means being responsible for a group of students. Thus, the problem of how a small number of adults can organize and manage a large number of students is the central institutional problem of schools. Furthermore, although there is enough social wealth, education has not been put first. The underlying aims of schooling seem to be to relieve the home of children for a few hours a day and to keep the kids quiet. Timid supervisors, bigoted administrators, and ignorant school board often inhibit real teaching. A commercially debouched popular culture makes learning disesteemed. The academic curriculum has been mangled by the demands of both reactionaries and liberals. Attention to each student is out of the question, and all the students—the bright, the average, and the dull—are systematically retarded one way or another, while the teacher's hands are tied. Naturally, the pay is low for the work is hard, useful, and of public concern. In spite of these conditions, teachers do not, for the most part, succumb to cynicism or indifference, the students are too immediate and real for teachers to become callous. However, given the conditions of schooling, can teachers fail to suffer first despair and then deep resignation? The resigned teacher sees little need for research-based knowledge to survive under these conditions.

Nevertheless, I believe most teachers want to act as and be treated as professionals, even if the working conditions in many schools make it nearly impossible. One of the pressing problems facing all of education is how we can change the professional status and qualifications of teachers.

IS "EDUCATIONAL" RESEARCH RELATED TO TEACHING?

The major purpose for doing research is to develop new knowledge about teaching and learning. The new knowledge is assumed to be valuable because it will lead eventually to the improvement of practice in classrooms. Jeremy Kilpatrick, in a recent paper titled "The Reasonable Ineffectiveness of Research in Mathematics Education" (1981), presented a number of reasons why current research in mathematics education has not been effective. Three of these reasons I would like to emphasize are lack of identity, lack of attention to theory, and our failure to involve teachers as participants.

Identity. Kilpatrick argues that "...most of the research studies in our field are conducted as part of the requirement for a doctorate and that most of these are done by people who will never do another piece of research" (p. 24). I would argue that most dissertations should be considered only as reseach exercises. Their purpose is to give graduate students a chance to learn how to conduct research. However, such research does not often arise from or contribute to a research program of a community of scholars.

Productive research happens when consensus occurs among a group of scholars about the legitimate problems and methods of research for a problem area. At that point, I would argue we can identify "research programs." It is from such programs, not individual studies, that implications for practice will be found. Elsewhere, I have argued that for problems like children's learning to count, add and subtract, or understand rational numbers, consensus is emerging (Romberg, 1983).

Theory. Kilpatrick also states "...it is only through a theoretical context that empirical research procedures and findings can be applied" (p. 25). Without giving serious attention to the conceptual frame of reference upon which the study is based, it si lifeless and extrapolation to practice is of little value. Furthermore, the choice of theoretical constructs has not been generative as Fran Schrag has argued (1981). "For too long, we in educaion have been concerned with the nature and foundations of knowledge rather than its uses" (p. 280). Robert Glaser (1976a, 1976b) has argued that a primary reason for the lack of success in applying theories of learning to instruction is that learning theories are descriptive whereas theory of instruction are necessarily prescriptive. Learning theories describe how children learn or think; instructional theories predice the effects of instruction. Prescriptive theories simply do not follow directly from descriptive theories. It should also be noted that productive research programs are theoretically based.

Involving Teachers. Finally, Kilpatrick has argued that one way of improving the effectiveness of research is "to involve teachers in our research" (p. 25). Only by moving research out of the laboratory and into the classroom, by developing dynamic theories of classroom instruction, and by making teachers partners in the effort will research-based knowledge be generated that will be truly useful.

The above argument does not mean that there is not useful research-based knowledge that is relevant to current practice. In a recent paper that Tom Carpenter and I have written (in press), we summarized knowledge from two disciplines, cogcnitive science and classroom teaching. We found that "current research is beginning to establish sufficient findings so that significant changes are called for in the teaching of mathematics" (p. 67). This is not the place to review the details of that argument. However, a major task of the next decade will be to bring the variety of constructs from both disciplines together and relate them to an appropriate view of the mathematics which should be taught.

THE POTENTIAL CONNECTION BETWEEN RESEARCH AND TEACHING NEEDS TO BE DEVELOPED

Alan Bishop (1977, 1982) has argued that teachers can borrow three things from researchers: their procedures, their data, and their constructs. Note that Bishop did not include "results" among the things to be borrowed from researchers. As Kilpatrick (1981) has argued, "Too many mathematics educators have the wrong idea about research.... They give a high priority to summarizing and disseminating research results so that teachers can understand them" (p. 27). A researcher makes a contribution to classroom instruction not by results, but by providing alternate constructs about teaching and learning, and methods and procedures of inquiry.

What I have tried to argue in the first part of this paper is that while the job of teaching at present cannot be called "professional", it could be; and that while most "educational" research is not very useful, it too could be. Thus, the problem is how to build a profitable connection so that research-based knowledge would be a basis for the judgments and decisions of mathematics teachers.

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THE UTILITY OF RESEARCH

David Wheeler, Concordia University
Canada

A statement about research in mathematics education adopted in September 1983 by the National Council of Teachers of Mathematics (USA) says, in part:

> "...if research supports the value of a particular teaching strategy, then the learner benefits; if research indicates that a particular instructional approach is more efficient than others, both the learner and the teacher benefit; if research suggests directions for program and policy decisions, then administrators, supervisors, and curriculum developers benefit. When research clarifies our understanding of the teaching and learning of mathematics, all people benefit."

A question this paper addresses is whether there is, or could be, such a simple and direct relationship between research and benefit as is suggested by the NCTM statement. I also give my views on where the real utility of research lies, and suggest some research directions which might make research more useful to teachers than it is.

THE BENEFITS OF RESEARCH

Not all that is done in the name of research is good research, or even true research at all. As Fruedenthal (1981) asked in his address to ICME4 ("Major problems in mathematics education"), how can we tell, especially in this time of increasing quantities of mathematics education research, the good from the bad? We haven't developed, in mathematics education, the established canons of truth and argument of a discipline like mathematics, or the informed public (however small) that can ensure that research results are scrutinized to make sure that the canons have been applied. We have only to read Gould's Mismeasure of man (1981) to appreciate that some researchers, under similarly unfettered conditions, will consciously or unconsciously interpret their research results with the bias of their preconceptions, or even pervert the research process altogether.

It does not do to be naive in this area. Consider a type of research model that is commonly employed in empirical studies - the pretest/treatment/posttest model. It is not unusual for the "treatment" to consist of, say, 10 hours of teaching time with a group of 20 students. What level of significance can we possibly expect from an experiment of that duration with a group of that size? Would we be able on the basis of such experiments to find adequate "support for the value of a particular teaching strategy" or an indication "that a particular instructional approach is more efficient than others?" Surely not.

The NCTM statement is, of course, a political statement, partly an attempt to be persuasive about the value of research. But it glosses over the substantial gap between what we might like research to be able to say and what it can actually at this moment say, and it ignores completely the problem of evaluating the quality of competing researches in the field. Do teachers and administrators (who are persumably being addressed) have to be talked to so paternalistically? Perhaps they already understand that educational questions are difficult and complex and that most research in the field is immature and inconclusive. I am not attacking the efforts or abilities of researchers, but trying to be clear about the state of the game. Even in established fields (which mathematics education is not, as I have said) research problems can be intractable. A cure for cancer would have been discovered decades ago if money, ambition, and talent were enough.

We need not, though, swing to the other extreme and dismiss research as having nothing to say to teachers, no benefits at all to bring. I shall not here discuss the potential contribution of particular researches since I believe other speakers will be doing that, but will open up - perhaps in a rather idiosyncratic way - the situation to a different picture of the function of research.

THE TRUE UTILITY OF RESEARCH

The perceived usefulness of research to teachers depends, in the main, on the extent to which they perceive, or are able to conceive, that they might teach differently. If they believe that they could not, or need not, then research will be an irrelevance, at best an intriguing intellectual exercise.

For some teachers, the constraints on their teaching that prevent them from changing are external. They would "like to" teach differently but they "cannot" - there is not enough time, they must follow the mandated curriculum, their principals will not let them, their students are not bright enough, and so on. I will not discuss this point further here, except to make the obvious remark that, yes, teachers in educational institutions are subject to constraints that are not of their own making, but that few such contraints are so powerful that they leave no room for menoeuvre. This is, though, a serious issue. Who can remain complacent while institutionalized education forces even a minority of teachers to teach less well than their best?

A less obvious constraint is the model of teaching that some teachers have internalized. In this model, the subject matter to be taught is already determined in content and form, the teacher knows this subject matter and passes it on, "as is", to the students, and the students rehearse it until they can show they know it as well as, or nearly as well as, their teacher. What place can there possibly be for research if this is the state of affairs?

Everyone knows that this simple model rarely works in the way it should, but often when it doesn't, the model is not abandoned, but only modified in structurally unimportant ways. For example, the subject matter is broken up into sequences of small pieces, recurrent difficulties are anticipated and prepared for, illustrations are selected which interest or "motivate" the students, and so on. None of this changes the basic presentation-rehearsal form of the classroom activity, and teachers do not have to change their role in any substantive way.

But what if the simple model is simple-minded, even crass? We may remember the example of Socrates' lesson with the slave-boy, described in Plato's "Meno", which does not take the presentation-rehearsal form. Indeed, Socrates does not believe that the subject matter, the mathematics, is "in" the teacher but not yet "in" the student, as the simplistic model supposes. He believes that it is already "in" the student (as well as the teacher), but that it has not yet been brought into consciousness. The teacher's job, on this view, is not to present the mathematics, since the student already knows it, but to cause the student to fetch it up from within himself so that he becomes aware that he knows it. Though few, if any, teachers can share Socrates' particular beliefs, the example is instructive in showing an alternative teaching model that appears to work at least well enough to indicate that the presentation-rehearsal model is not the only contender. The example also shows (as those of us who admire it must admit) that a teaching model may work even when the beliefs that inspire it are mistaken.

In this case, maybe we can formulate a different and more acceptable set of assumptions that would explain the success of the socratic method. Perhaps the significance of the teaching style described in the "Meno" is that, in effect, it makes the student construct his own mathematical knowledge—construct it from things already known, plus new information and hints, sorted out and combined through a process of personal experiment. The teacher is not an "instructor" (hateful word) but someone who urges, prompts, and validates the activities of the student. The text seems to me to support this interpretation in detail better than it supports Socrates' own story about the rationale of his method.

The power of this example, I suggest, and the reason why after more than two millenia many readers return to it for inspiration, is that it "deconstructs" everyone's naive preconceptions of what a teacher is supposed to do. Here is a teacher who doesn't simply take the student through what is to be learned, who deliberately arranges the lesson so that the student may make crucial mistakes, who doesn't tell the student when he is wrong, who tells (almost) nothing and asks (almost) everything. The story offers us the paradox, which I hope we can still permit to disturb us, of a teacher who "does not teach" (as Socrates himself expresses it) although the student clearly learns by reason of what the teacher does.

Students are adept learners. In their very early years, they organize their own learning. They acquire speech, and social and physical skills, by picking out what they need from the environment, by attending to the feedback provided by their own bodies and the people and objects around them, and by practicising assiduously. Their learning skills do not desert them as they grow older as anyone can see by watching how they continue to learn things that no one is trying to teach them. When they do not display these learning skills in the classroom, we ought to ask why they do not (and not, God forbid, how we can "teach" them the organizing skills they appear to be lacking). Perhaps we should entertain the possibility that what happens in the classroom may be inhibiting the application of their skills.

Mathematics in books, even in most textbooks supposedly written for learners (nearly all mathematics books being textbooks for someone or other), shows what the learner should be able to do and understand when he or she "arrives", when he or she has mastered the contents. They rarely show, although they may make the attempt, what the learner has to do "on the way". Recall, for a moment, the differences between the course of the slave-boy's lesson and the way a textbook might present the mathematics that he learned. It may be that it is impossible—even undesirable, since it would stereotype a spontaneous event—for a textbook writer to follow the same course as Socrates and the boy. The most interesting question that then arises, a question central to pedagogy, is: how does the teacher construct a lesson that will allow the students to construct the mathematics, given that the textbook describes the mathematical destination, not the journey?

One requirement is that the teacher be able to "deconstruct" the mathematics of the textbook. By this I mean more than "breaking it down", which is the only form of deconstruction commonly recognized, though it is a start. I mean the experimental process by which the teacher investigates the mathematical content, abandoning preconceptions about how it is put together, where it fits in a sequence of topics, what has to be understood before it can be learned, and so on. A detailed description of the process would take too long (and is, in any case, and subject for more study, as I suggest in the next section), so I must leave the reader tantalized and dissatisfied at this point. But the end result of the process is that the teacher has created some alternative possibilities for the construction of the particular piece of mathematics. Now he or she is in a position to choose a starting point and knows enough about the mathematics involved to be able to construct a lesson around the students' responses, freeing them from the constraint of having to follow one well-trodden path.

In this section, I have indicated two areas where it is undesirable that we "deconstruct" our usual assumptions--the functions of a teacher and the form of mathematics presented for learning. The two are interconnected, as they are with other areas also requiring deconstruction. But perhaps I may now put my main theme in a different way and express it as "the reconstruction of commonsense".

Commonsense is not fixed. At various times, it was commonsense for people to hold that the earth is flat, that the sun rotates around the earth, that the blood in the body ebbs and flows, that substances lose part of themselves to the air when they burn. If our commonsense about these phenomena is now different, it is because research has forced us to change our common assumptions to more correct ones, and to more useful ones. I believe our commonsense about teaching and about mathematics needs reconstruction too. The ultimate utility of research is that it forces us to abandon unexamined assumptions and to reconstruct our commonsense to make it correspond better to the behavior of the phenomena with which we must deal. And what could be more useful than that?

WHAT RESEARCH?

In this section, I suggest three areas where more research is needed, or research of a different kind from much that is being done, if we want research to be as useful as possible to teachers of mathematics. The three areas are closely related and connect with the concerns I have expressed in the previous section.

Mathematics. I take it as obvious that a teacher should know what he or she has to teach, but that this is not enough. What more is required? More mathematics? Perhaps, and some people say so. But more important, for a teacher, is to know more things about mathematics, those things that a mathematician doesn't have to bother with. For example: all the mathematical "know-hows" that a mathematician uses in addition to mathematical knwoledge: how to attack a new problem, how to try to prove a result, how to organize a search for a useful technique, how to generalize and specialize, how to take a piece of known mathematics apart. Further: the different modes of mathematical thinking, the use of induction, deduction and mental imagery, the characteristics of mathematical language, the difference between clarity and precision, mathematical metaphor. Again: the way that mathematical concepts develop, the roles of action and perception in mathematical activity, the parts played by intuition and logic. And: mathematics as a human endeavor, with a history and a sociology.

This is by no means an exhaustive list - it omits any philosophy of origins and technology of applications, for instance - indicates how much more there is to know about mathematics than mathematics itself. No one, of course, will know all there is to know about mathematics, and few teachers will need more than an elementary acquaintance with the items on my list. But is it not a clear responsibility of professional training to concern itself with these issues?

Research into some of these aspects of mathematics goes on, much of it undertaken by people who are not educators. It seems peculiar, in a way, that those who stand to gain most from research into these aspects have not much involved themselves. The reason, perhaps, is that educators see their problems everywhere but in mathematics itself and so take it as a "given". The desirability of breaking into this circularity makes me put this aspect up front.

Teaching. In the past 2500 years, we have found out a great deal about how people, including children, think and learn, but we know hardly more than Socrates did about teaching. Researchers have behaved as if learning were harder to understand than teaching. On the contrary, teaching is much the more mysterious activity. We know almost nothing about why some teachers are "better", in various senses, than others, and we don't really know how to help teachers to become better.

We are in this repsect no worse off than other professions, which also prepare their entrants "by guess and by God", but we cannot afford this ignorance. Our profession is numerically the largest and the one with the most varied human intake. Our profession is also education. How can we profess it if we do not know how to educate our own members?

Knowing. In human life, "knowing" seems more pervasive that "knowledge"; it relates to more of our inner and outer worlds than knowledge can reach. We might say, for example, that we know how to stand and walk and speak; we can say, and wo do say, that we know how to play a game or a musical instrument. These knowings are chiefly skills; but we can also say that we know a person, a place, or a picture, which knowings are not skills and not what we usually call knowledge either. Some knowings are almost coextensive with knowledge, as when we say we know some mathematics or some history. Other knowings transcend knowledge as when we say that we know a sculpture is beautiful, a proof elegant, or a law humane.

From this point of view, knwoledge takes its place as a sort of sediment, a precipitation, from those knowings that are, firstly, verbalisable, and secondly, regarded, by someone or some group, as worth preserving and accumulating. Knowledge is, indeed, frequently found in books, which are the traditional means of preserving it, but it remains inert until it is repossessed by persons who can integrate it with what they know, converting it back into knowing.

Epistemology (the theory of the method and grounds of knowledge, according to the Concise Oxford Dictionary) could be developed without difficulty to become a means of studying knowing rather than knowledge. Piaget called his main work "genetic epistemology" and showed that an epistemological method can handle the development of knowledge in children. It does not seem too much to hope that future researchers will extend the method and devise an epistomology of "coming to know" which could serve all educators.

CONCLUSION

This paper may take too jaundiced a view of the utility of the research that has been done in mathematics education and too optimistic a view of the potential utility of future research. Other papers by other people will no doubt strike a different balance. But research partakes of this duality. The researcher must believe that even the most difficult questions may be answerable, but he also knows that, however significant his findings they will inevitably open up new areas of ignorance. We never know all that we would like to know, or all it would be useful to know, but it is open to all of us--researchers, teachers, students--to claim a little new territory, if only for ourselves.

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REVIEW AND DISSEMINATION OF RESEARCH[1]

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Research is frequently an activity that proceeds independently of the utilization of the fruits of that activity. This dichotomy can be referred to as "pure research" versus "applied research" or "research" versus "application." Whatever the references may be, the value of research can be fully realized only if the research and application processes are brought into harmonious relationship. Short summarized this relationship between "knowledge production" and "knowledge utilization" in the following way: "Ultimately, the resolution of practical issues depends upon improved coordination between the process of knowledge production and the process of knowledge utilization. For the entire process to proceed optimally, its major features must be understood and the points of possible breakdown recognized and overcome" (Short, 1973, p. 237).

The process of research has been studied in great detail. The advances in research designs are well known to those in the field. But the process of research review and dissemination is not well known. Jackson summarized this lack of understanding "...one might expect a fairly well-developed literature on methods, techniques, and procedures for conducting (research) reviews, but this is not the case. An earlier examination by this author of a convenience sample of 39 books on general methdology in sociological, psychological, and educational research revealed very little explanation..." (1980, p. 438).

One might hope that researchers would provide solutions to the dissemination-application problem. But hopes for such a solution are not well founded as researchers infrequently assume this task. Kerlinger commented on this in the following way:

> The researcher is preoccupied with, and should be preoccupied with, variables and their relations. He should never be rquired to think about or to spell out the educational implications of what he is doing or has done. To require this is to require a leap from an abstract relational level of discourse to a much more concrete and specific level. This cannot be done directly; it is not possible

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> to do a research study and then have practitioners immediately sue the results (Kerlinger, 1977, p. 6).

When applied to mathematics education, Fennema (1981, p. ix), in her introduction to Mathematics Education Research: Implications for the 80s, observed that research often does not find direct application in the classroom. She said:

> Missing from this list of contributions of mathematics education research is any mention of providing information that will tell a mathematics teacher, at any level, what to do in her or his classroom. This is a deliberate omission because I firmly believe research cannot give precise directions to what a specific teacher should do in a particular classroom. This is not to say that research is not helpful to the classroom teachers. It is only that research cannot, nor should it even if it could, tell teachers exactly what they should be doing as they plan, conduct, and evaluate instruction.

So this is our program, the process of research proceeds independently of the processes of application and dissemination. The process of research is well-known, the process of application and dissemination is not. This paper and the Using Research Group of the Congress will address the issues of reviews and dissemination of research. Hopefully, we will make progress to provide an impetus to establishing a harmonious relationship between the processes of knowledge production and the process of knowledge utilization.

THE REVIEW OF RESEARCH

At the heart of dissemination is the review of research. The construction of a review involves several phases or tasks. These tasks have been identified and discussed by Cooper (1982), Jackson (1980), and Ladas (1980). The specific tasks vary from one reviewer to another, but basically, they consist of the following: (1) the selection of the broad areas or topics of review, (2) locating the studies to be reviewed, (3) representing the design characteristics of the studies and their findings, (4) analyzing and integrating the findings of several studies, (5) interpreting the results for practitioners, and (6) communicating the review. In the following sections, each of these tasks will be discussed.

SELECTION OF REVIEW AREAS

The selection of an area for review is often pragmatic; that is, the author of the review is responding to a request from an editor or a need of practitioners. In addition, one hopes that a review can satisfy a theoretical need. Jackson (1980) recommended that four sources be consulted in the review process:

(1) the available theory structure which may suggest questions for review; (2) prior research and particularly any research review that may have been written on the topic; (3) the primary research that is to be reviewed; and (4) the reviewer's own insight, ingenuity, and intution as to which topics are ripe for review.

The review might be very broad; such as mathematics in the elementary school or mathematics in the secondary school (e.g., Dessart, 1964; Dessart & Frandsen, 1973; Driscoll, 1981; Riedesel & Burns, 1973; Romberg, 1969); it may cover a year's research (e.g., Dessart & Burns, 1967); it may deal with a single subject; such as algebra or geometry (e.g., Dessart & Suydam, 1983); it may cover a specific skill; such as, computational skills (e.g., Suydam & Dessart, 1976); or it may discuss a single topic; such as, manipulative materials (e.g., Driscoll, 1984; Suydam, 1984).

LOCATING THE RESEARCH STUDIES

Once the broad area of review has been determined whether it be, for example, elementary mathematics, algebra instruction, computational skills, or manipulative materials, the research studies must be located. In early reviews (1950s and early 1960s) a painstaking search of individual periodical indexes often proved to be the most certain method of finding relevant studies. Quite naturally, this proved to be a time consuming task. Very fortunately, two recent advances have significantly improved the reviewer's task of locating studies.

The first of these is a computer search through Education Resources Information Center (ERIC) files. By using key words and logical connectives of those words, one can find the titles of many relevant studies. A second significant development is the annual publication of the annotated bibliographies by Suydam and Weaver (1971 through 1984) in the Journal for Research in Mathematics Education. These bibliographies consist of titles systematically collected from over 80 journals (virtually all relevant American journals and also such journals as the Alberta Journal of Educational Research, the Australian Mathematics Teacher, the British Journal of Educational Psychology, and Educational Studies in Mathematics). While most of the journals consulted are written in English, one can speculate that in the future, resources will be made available to conduct searches of non-English journals as well.

In a search, those studies that have serious methodological flaws should be eliminated from further review. With current editorial policies, studies with serious flaws are published infrequently. While the reviewer hopes to eliminate studies with serious flaws, there is another inherent bias for which the reviewer has virtually no control. Greenwald (1975) observed that about fifty percent of researchers who rejected a null hyptohesis submitted their work for publication, whereas only a small percent (about five) of those who failed to reject would attempt publication. If this generalization is true for all researchers, then those studies that reach the reviewer are those in which the experimental treatment has proved useful. This means that the reviewer must temper his or her remarks and insights to accommodate this limitation.

Since the review area may be broad at the outset of the search, the reviewer will find that it will be necessary to partition the complete set of studies into categories dealing with the same topic (e.g., computer assisted instruction in ninth-grade algebra). Once this is completed, the number of studies may be small (less than 10), so that the reviewer can carefully examine each of the studies. In the event that the number of studies is large (25-50), it may be desirable to select a random sample of, perhaps, 10 studies for a more complete analysis. In any event, the reviewer should report the methods used in the search, the bibliographies consulted, and the details of the search. The reader can better judge the merits of the review, when such information is made available.

DESCRIBING THE STUDIES

Following the assembling of the body of studies to be reviewed and the partitioning of those studies into subtopics, the reviewer faces the task of describing the studies and representing their characteristics. The reviewer must decide the extent of the description that he or she wishes to include. This decision depends upon the audience of the review. If the audience is primarily researchers, then the reviewer may prefer to emphasize technical aspects including details of the hypotheses, design features, sample sizes, types of statistical tests, and conconsions. On the other hand, if the audience of the review is the practitioner, then the implications of the study for practice may be emphasized to the exclusion of more technical details.

Because the amount of detail included in the review is a function of the needs of the audience, it will vary considerably from audiences of researchers to those of practitioners. In the reviews by Dessart and Suydam (1983) and

Suydam and Dessart (1976), the emphasis was upon capturing "ideas" that would be useful to practitioners. This point of view appears to be consistent with the observation of Romberg (1985) who noted that Bishop and Kilpatrick argued that the most significant contributions that researchers can make to teachers are not necessarily "results" but rather "constructs" about teaching and learning. In a similar vein, Baker (1984, p. 455) urged that researchers should influence practitioners by conducting research that can "provide a rich source for generating new ideas, hypotheses, and even theories." These viewpoints seem to be in substantial agreement with Dessart and Suydam (1983) who advocated that it is the "ideas" of research that prove more useful to the teacher than results. They further encourage teachers to "try ideas (from research) in their classrooms, retaining those that prove useful and rejecting those that do not" (Dessart & Suydam, 1983, p. v).

ANALYZING AND INTEGRATING FINDINGS OF STUDIES

In spite of the philosophical differences between emphasizing "results" or "ideas" in a review, the task of analyzing and integrating the findings of studies is substantial. If the number of studies on a topic is small (3-5), the task of the reviewer is one of describing the preponderance of positive or negative findings and drawing conclusions from that preponderance. If the number of studies is large (10-25 or more), then the mental task of ascertaining a preponderance is not easy. More sophisticated methods are needed.

One such method is suggested by Jackson (1980, p. 446). He reprsents findings as significant (+), nonsignificant (+), zero (neutral), nonsignificant (-), and significant (-). The "significant (+) and significant (-)" refer to statistically significant findings favoring or not favoring the treatment; "nonsignificant (+) and nonsignificant (-)" refer to non-statistically significant findings favoring or not favoring the treatment and, the "zero" or "neutral" category refers to studies in which no preference was found either "for" or "against" the treatment.

This "box-score" or "vote-counting" method, although an attempt to introduce quantification in the integration of findings, has weaknesses. For example, it does not distinguish between studies that are "significant" ($p < .05$) and those that are "highly significant" ($p < .01$). Consequently, the question of falsely rejecting the true null hypothesis is not considered in the integration process.

Furthermore, such a box-score comparison is only valid if the dependent variables are, in fact, identical in all treatments. Ladas (1980, p. 602) found that reviewers claimed that studies were not providing support for a certain variable, e.g., "note taking", when the variable was in fact, "note taking with review or without review." In regard to the box-score method, he wrote: "This analysis shows a tendency for the condensation process to result in a blurring of details, overgeneralization, or even misrepresentation" (Ladas, 1980, p. 602).

Another method involves the cumulation of p-values across studies. An overall p-value is determined for the entire body of studies; much as if, the complete set of studies had constituted a single study. This procedure produces a simplistic result, that is, an overall p-value, but often times the magnitude of the effect is left unknown. However, there are statistical procedures for testing the statistical significance of the p-value and for estimating the effect sizes if the sample sizes are known (Hedges & Olkin, 1980).

The most powerful methods of cumulating research findings across studies fall under the rubric of meta-analyses. Two of these analyses, the Glass Method and the Schmidt-Hunter Method, place a strong emphasis upon the notion of effect size, which is the difference between the treatment and control means divided by the standard deviation of the control group of a pooled standard deviation. From these effect sizes, a mean and standard deviation of cumulated effect sizes are found. These prove to be useful measures of comparison. In the Glass Method, the variance of effect sizes is also studied at face value for substantive explanations (Hunter, Schmidt, & Jackson, p. 138). The Schmidt-Hunter Method analyzes the variance of effect sizes further to determine if they were due to various statistical artifacts; such as, sampling error, reliability of measures of independent and dependent variables, various range restrictions, instrument validity, and other factors.

The box-score method, the overall p-value procedure, and the Glass-Schmidt-Hunter methods represent advances when rigorous methods of summarizing large numbers of studies are necessary. Fortunately or unfortunately, in mathematics education, the number of studies across any one variable is often small. But if one adopts the review stance that it is the constructs, the teaching procedures, and methods revealed by research that are valuable to the practitioner, then the rigor of the integration methods is important but certainly not an overriding consideration. While these methods of integrating the findings of studies are useful, they do assume that the set of studies is, in fact, dealing with the same phenomenon. Often times this may not be the case. Consider the term "discovery learning." It may have vastly different meanings from one researcher to another. In one case, it may imply virtually no teacher intervention and in another, varying amounts of teacher involvement in the learning process. Consequently, one may find very substantial differences in results of studies that can be classified as "discovery learning" studies.

INTERPRETING RESEARCH RESULTS

Interpreting the results of research may have several goals. Among these are: (1) confirming old and accepted theories, (2) disproving or lessening the credibility of old theories, (3) formulating or suggesting new theories, (4) providing directions for future research efforts, (5) providing directions for future reviews, (6) formulating recommendations for policy or practice, and (7) summarizing teaching methods, constructs and ideas for use by practitioners in teaching situations. The first five of these goals are useful for both the practitioner and the researcher, but are probably more valuable for the researcher. On the other hand, the last two: formulating recommendations for practice and summarizing teaching methods and constructs will be more useful for the practitioner.

Dessart and Suydam (1983) empahsized these latter aims in their work. The technique was to capture succinct ideas in a series of short statements which were enclosed in rectangles spaced throughout the body of the text of the review. These statements or ideas dealt with descriptions of methods and policy recommendations with some notions as to their past success as revealed by research. Practitioners were cautioned not to accept these as "universal truths" but rather "ideas" that they may wish to try in their classrooms (Dessart & Suydam, p. 56). Dessart and Suydam (1983) applied three broad criteria to the selection of those ideas that would be highlighted in the publication. These criteria were: (1) the idea must be useful to school practitioners, (2) the idea must be stated succinctly and unencunmbered by technical details and jargon, and (3) the idea should be supported by valid research.

DISSEMINATING THE REVIEW

The communication of the review to the field to both researchers and practitioners is the most important part of the review process. It is through this communication that the research has some promise of reaching the classroom where its effects may improve instruction. One may classify the communication process into three broad categories: (1) publications, (2) meetings and conferences, and (3) activities of higher education. Publications have represented the most useful means of communication in the pat, but reports at conferences and various means of dissemination used in higher education are becoming more prevalent.

Publications can be grouped into major works that are published periodically, special monographs designed to address special issues, and journal reports. Major works such as the Review of Educational Research and the Handbook of Research on Teaching, published by the American Educational Research Association (AERA), appear periodically. The Handbook, published every 5-8 years, takes a very broad look at mathematics education at the elementary and secondary school levels. The Review of Educational Research tends to explore topics in more depth but deals with mathematics on a very occasional basis. The most recently published report in mathematics was in 1976. It was an update on attitudes and other affective variables in learning mathematics (Aiken, 1976). Monographs represent a ready means of publishing research reviews on special topics. Classroom Ideas From Research on Computational Skills and Classroom Ideas From Research on Secondary School Mathematics, previously discussed in this paper, in addition to Elementary School Mathematics: A Guide to Current Research by Glennon and Callahan (1975) and Research Within Reach: Secondary School Mathematics by Driscoll (1982) represent this type of reporting. Journals, such as the Arithmetic Teacher, The Mathematics Teacher, The British Journal of Educational Psychology, and others represent means of bringing research reviews to the teaching public rapidly and efficiently. Such journals have experimented with reviews from time to time but have not developed a consistent pattern of delivery. The reasons for this inconsistency have not been systematically investigated, but one can speculate that the lack of reader interest and satisfaction have been the primary reasons.

Meetings and conferences provide other avenues of research dissemination. Meetings can vary in length from hour-long sessions as frequently presented in NCTM conferences to three or four day sessions as recently conducted by CEMREL, Inc., to disseminate the work of its "Research Within Reach Project"

(R & D Interpretation Service, CEMREL, Inc., 3120 59th Street, St. Louis, MO 63139). Conferences such as the International Congress on Mathematical Education provide an excellent way to disseminate research results worldwide.

Finally, higher education can serve a special role. Classes, seminars, discussions involving preservice and inservice teachers in which research topics are discussed along with their implications for the classroom is an ideal way to disseminate research ideas to the teaching community. Textbooks for teachers in which research ideas are integrated into the discussions or in which special chapters are devoted to research provide valuable ways of making research known to practitioners.

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PREPARING MATERIALS

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This paper deals with the role of research related to one component of the curriculum, i.e. the development or preparation of curriculum materials in mathematics intended for classroom use. Following Tyler (1967) and Romberg (1970) research will be used to include elemental (basic) and evaluative research. It is assumed that the development of curricular materials in mathematics is moderated by the degree of teacher participation in such development. Consequently, the role of research in the development of curricular materials will be discussed within the framework of three curriculum models with increasing level of teacher participation: Research, development and diffusion (RDD) model; problem-solving model; and social interaction model.

The thesis of the present paper is that the conclusions of research have had limited and isolated impact on the development of curricular materials. However, research constructs and procedures, if described and interpreted in context, have a potential of being used in the development of curricular materials. This potential increases with the increase in the level of teacher participation in such development. The rest of this paper will be devoted to the presentation and illustration of this thesis.

THE INTERACTIVE NATURE OF MATERIALS PREPARATION

The question of developing curricular materials is essentially an interactive one. Normally, and except for few simple cases, the question takes the form of at least a five-fold interaction as suggested by Shulman (1970). Rephrased for the particular context of curriculum materials development, the interactive question becomes:

> For a group of learners with known characteristics, what type of presentation (degree of teacher control and sequence of activities) of this mathematical task (concept, generalization, skill), and in what amount (instructional time), are needed to produce a specified pattern of responses?

Research on interactions focused mainly on two-way interactions (ATI) between instruction and aptitude (any characteristic of the person that affects his response to the treatment [Cronbach, 1975]). The complexity of higher order interactions is prohivitive. It becomes more so when one attempts to investigate all possible interactions among factors related to the entering characteristics of students, type of instruction, amount of instruction, mathematics subject matter, and nature of learning outcomes. The hybrid suggsted by Cronbach as a result of crossbreeding experimental and correlational research does not seem to offer sufficient help for an individual or a group of individuals engaged in the development of materials.

THE SITUATION-SPECIFIC NATURE OF MATERIALS PREPARATION

In developing classroom materials, an author is not only conscious of five-fold interactions, but also of the situation in which the materials are to be used. The situation includes, among other things, interactions such as those of teacher-student, student-student, student-parent, and a host of other higher-order interactions, all within an established system of beliefs and values. To cope with this complexity, the author is bound, consciously or unconsciously, to reduce the situational aspects to global conceptions. The effectiveness of such conceptions is a major factor in the effectiveness of the developed materials. In other words, although mathematics education deals with universal problem areas, the solutions for such problems are far from being universal (Christiansen & Wilson, 1974). The universality of mathematics as a discipline is no guarantee for the universality of mathematics education.

What do research conclusions offer in this respect? The so-called "conclusions" of research are no more than credible hypotheses which apply to the situations in which the studies were conducted. It is difficult to imagine, for example, how the conclusions of a research conducted in a western country even if under highly stringent controls can apply to other countries. Students who are expected to memorize, by age eight, a whole book required by culture or religion, or who learn mathematics in a foreign language acquire aptitudes different from others of the same age and developmental level.

PROBLEM OF CONTROVERSY

It is not unusual in research to arrive at contradicting conclusions even when studies are replicated under more or less similar conditions, indicating the importance of the role which situational variables play. Many of these contradictions may also be accounted for in terms of differences in constructs used; and, research procedures. A case in question is the effect of degree of guidance and sequence of instruction on achievement (known as the discovery-expository controversy). Questions of essential importance to the development of materials are often either not attempted or not answered in a reasonably definitive manner. Examples of such pertinent questions include: How are basic skills developed effectively? How much emphasis should one give to operations in developing algorithms? How could problem solving in mathematics be developed? Research conclusions are too controversial to be of great help in this respect.

PROBLEM OF COMMUNICATION

If it is true that research conclusions may be accounted for in terms of idiosyncracies more than regularities, then it would be more useful to the developer of curriculum materials to know the conditions and contraints under which the research was conducted. Unfortunately, empirical research reports in mathematics education are modeled after similar reports in social science. Such reports according to Freudenthal (1979) "are little more than short summaries, or in the most favorable cases, abridged versions of voluminous reports, inaccessible to outsiders even if they still exist after a lapse of five or ten years" (p. 276). In particular, descriptions of research constructs and procedures, the most useful parts of a research report for materials preparation, are often sketchy.

RESEARCH AND CURRICULUM MODELS

The relationship between research and curricular decisions is moderated by the level of teacher participation in such decisions. The RDD model, a well known model in areas such as medicine, technology, industry, etc., assumes that findings of research can be used as a basis for development of a product (curriculum) to be used by the consumer (a teacher or student or both) according to specifications which are provided. The underlying assumption in the RDD model is that conclusions can be found across different situational and teacher variables and their interactions. As a result, my preceding remarks about the role of research in the development of curriculum materials apply here. An additional problem is that of evaluation. In education the product developed on the basis of research is a change of behavior, a new pattern of interaction (Eden & Tamir, 1979). Kilpatrick (1979) says that evaluation of a textbook, "is not like a refrigerator or a car. It cannot be warranted to perform as specified in every school or classroom in the same way it performed under laboratory conditions. The effectiveness of a textbook in promoting learning is highly situational, greatly influenced by the teacher who uses it, the pupils who study from it and the instructional setting in which it is used"

(p. 163).

The models of Social Interaction and Problem Solving give great attention to the needs, activities, and involvement of the teacher in the curriculum development process. In the model of Social Interaction, the curriculum group (specialists, researchers and resources) selects and develops samples of materials designated for teachers to be used in preparing learning materials compatiable with their students' abilities and needs. The Nuffield Mathematics Project is an example of the model of Social Interction. In the Problem-Solving model, the curriculum group coordinates, administers, and disseminates curriculum materials designated for students and prepared by various teachers. The role of research in both models is not as clear as in the RDD model. However, the assumed role of research in the models of Social Interaction and Problem Solving is to provide guiding principles to be optimized by the teacher according to the situation in which they are to be applied. Attractive as they may be from a research point of view, the two latter models assume a certain level of teachers' interest, initiative and professional development which, if not sustained, will lead to stagnation.

CONCLUDING REMARKS

It is most unlikely that the choice of a curriculum development model be made on the basis of the role of research in curricular development. Curricular decisions are deeply rooted in the social-political system of the community. It seems futile to think of any model as better than others. Instead we should try to make research findings more relevant and accessible to users and consumers of instruction irrespective of the model in use. This calls for a rethinking of the purpose of research whose ultimate aim is the improvement of actual classroom instruction. Cronbach (1975) called for reversing priorities by not "making generalization the ruling consideration in our research" (p. 124). In trying to observe, describe, and account for events in a particular setting, the researcher is to "give attention to whatever variables were controlled, but he will give equally careful attention to uncontrolled conditions, to personal characteristics, and to events that occurred during treatment and measurement. As he goes from situation to situation, his first taks is to describe and interpret the effect anew in each locale of series of events" (pp. 124-125). Generalization will come much later as results accumulate across situations. Even then a generalization is a working hypothesis, not a conclusion.

A second consideration is related to the research context. The classroom ought to be the arena on which the research endeavor is to be performed and won for the benefit of the teacher. A closer relationship is to be worked out among the teacher, researcher, and curriculum developer. To collapse the three roles in one person would be ideal but not practical. Alternative models have been suggested. Hawkins (1973) suggested that problems of education "are too long-term and too complex for the laboratory, and too diverse and non-linear for the comparative method. They require longitudinal study of individuals, with intervention, a dependent variable, dependent upon close diagnostic observation. The investigator who can do that and will do it is, after all, rather like what I have called a teacher" (p. 135). Cobb and Steffee (1983) suggest that researchers should act as teachers to ensure that models which we construct to represent our understanding of children's mathematical realities do reflect the teachers' understanding of children. Although teachers cannot be expected to develop all classroom materials, they ought to be trained to be "response sensitive" i.e. to be able to monitor the responses of their students to the developed instructional materials and adapt the latter to fit the needs and constraints of the specific situation.

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THE TEACHER'S VIEW AND THE TEACHERS' VIEW

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Professor Romberg has adopted the approach in addressing the problem of the use of research that the professional life of a teacher can be compared to that of a doctor. I would like to illuminate that analogy further by referring to the work of the one physician whose research dominated all doctors' actions for over one thousand years.

Galen, the Prince of Physicians, wrote more than three hundred books on human anatomy and physiology in the second century. He was the last great doctor before the Dark Ages and his teachings lasted throughout that time. In fact, his reputation grew so that even in 1559 when Dr. John Geynes stated that Galen's writings contained errors, the College of Physicians in London made him apologise - one wonders to whom! But Galen did have two big problems.

> "First, he was not allowed to cut up, or dissect, human bodies. Galen's theories about human anatomy came from what he saw when he cut up pigs, monkeys and other creatures. He even cut up an elephant and a hippopotomas! No wonder a few of his ideas were wrong. Galen's second problem was this. He was too sure of himself. He ignored facts that did not fit in with this theories" (Stevens, 1978, p. 6).

THE TEACHER'S VIEW

This statement about Galen's anatomical research reflects something of my own feelings about educational research. The school classroom is a complex animal (beast would be a better word at times), and I believe that any statement about how it "is" or "should be" must be treated with great caution because researchers by necessity must study isolated fragments of the creature.

But this does not mean that research results should be ingored. In fact, it is often the nonemotionally involved observer who can see both problems and solutions more readily. Rather it means to me that research results are only the first step in improving educational practice (which presumably is their aim). Unlike the doctor whose research base is in itself an improvement in medical practice, the teacher will always be the continuer and, in a sense, completer of educational research. For the teacher must apply the research result (gathered from what was in some way a fragmented study) to the complex animal. Each classroom being different, therefore, the same result could be successfully applied in one classroom, adapted in another and discarded in yet another.

The discussion above, of course, presumes that teachers are aware of educational research results. Therein lies another problem. Teachers are exposed to educational research discussions in their training, but thereafter, do not have the time to be reading deeply in the area. Consequently, the arguing between researchers about the validity or reliability of a piece of research which does go on is a good way of making teachers tune out. Such discussion should be kept between researchers. Just let us know when you've got it right!

Then when results have been widely enough agreed upon to be worth reporting, they need to be communicated succinctly and attractively. Generally speaking, teachers don't have the time to digest the academic details of research. They need to know the results and any important restrictions on their implementation. If it implies that teachers should regularly be retrained so that they do have the time to join the research discussion, then I also would see that as useful.

But these are my personal opinions and although I was asked to speak about the teacher's view of research, I don't really think my views on this topic are worthy of such exposure. I may well be some distance removed from the teachers' view. Accordingly, I have made a small attempt to try to ascertain....

THE TEACHERS' VIEW

I first turned to a survey carried out by the Victorian Secondary Mathematics Curriculum Committee in 1983. It was conducted personally by members of the committee during discussion with individuals or small groups in a range of post-primary schools. There were over 150 responses.

Part of the survey asked maths teachers to indicate their needs as follows:

SECONDARY MATHEMATICS CURRICULUM COMMITTEE NOVEMBER 1983

NEEDS

Please indicate your 3 main needs 'A' and indicate your 3 least needs 'Z'.

What we need most of all from the Mathematics Curriculum Committee are:

*Teaching ideas ☐
*Newsletters ☐
*Research projects and reports ☐
*Reviews of textbooks and other teaching resources ☐
*Inservice courses and conferences ☐
*Support for school-based inservice education ☐

*Curriculum development guidelines ☐

*Detailed course outlines including sample lessons ☐

*Guidelines for evaluation of programs and of student performance ☐

*Journal ☐

*Other (do not rate 'A' or 'Z') ☐

____

____

The research projects statement was rated A by 28 respondents and Z by 102. This rating only outranked "newsletters" and "journals" with which our service is already well supplied.

Next, I decided to conduct a survey of my own which was directed towards obtaining more detail of the teachers' view of research. Respondents were asked to rank their response to each of the statements below as either

A. STRONGLY AGREE
B. AGREE
C. NO OPINION
D. DISAGREE
E. STRONGLY DISAGREE

1. I use the results of Educational Research a lot in my teaching.

2. Some results of Educational REsearch which I have read have been absorbed into my teaching style.

3. I have learnt very little from Educational REsearch.

4. The most useful form of Educational Research is school trialling of materials.

5. Educational Research tells me nothing I didn't already know.

6. I feel many of the findings of Educational REsearch are interesting but not applicable in my classroom.

7. The results of Educational Research are difficult to apply because they are derived in special circumstances.

8. Results of Educational REsearch are not communicated in an easily readable and understandable form.

9. Educational Researchers' are always contradictory each other's findings.

10. Classroom teachers carry out their own research day by day.

11. I have more cnfidence in the advice of an experienced teacher than the findings of Educational Research.

12. I am more likely to apply my own experience than the findings of Educational Research when solving a problem in my classroom.

13. If a piece of Educational Research suggested marked changes in teaching practice, others may change but I probably wouldn't.

14. If Educational Research had never begun, my teaching today would still be much the same.

15. The main purpose of Educational Research seems to be to keep Educational Researchers in a job.

M.1. Maths is one area where I am aware of the value of research findings.

M.2. The work of Piaget, Dienes, and others has had a discernible influence on mathematics curriculum and teaching.

M.3. The work of Piaget, Dienes, and others has had a discernible influence on MY mathematics teaching.

M.4. I need to know more about any results of Educational Research in the mathematics area which have been accepted as correct.

M.5. The best way to teach mathematics is with plenty of practice exercises and no research can improve that.

The survey was conducted in my own school and, through the courtesy of some colleagues, in a Victorian high school and two other primary schools (one is Scotland and one in the USA). The teachers involved reprsented a wide variety of years of experience and between them taught all classes from preparatory to year 12. Some administrators were also included.

Between the primary schools, there were ~120 teachers involved, ~100 of whom currently taught mathematics. The majority of teachers were from Australia, but approximately 12 Americans and 12 Scottish teachers were surveyed.

I decided not to include the high school results in the discussion which follows. Perhaps I'm doing a Galen. My reasons were that only nineteen members of a staff of 75 found time to answer. Therefore, the sample may not be representative of general opinion. Of the nineteen who responded, five forgot to turn over their sheet to answer questions 10-15 and M1-M5. This perhaps indicates that some teachers were under too much pressure to give the statements adequate consideration. Finally, only six of the nineteen currently taught mathematics. However, I have included the numbers responding in each category in the chart below, so that you may decide for youself whether they should have been included.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	M1	M2	M3	M4	M5
A	1	1	0	1	0	0	0	2	2	0	1	4	0	0	2	0	0	0	1	0
B	5	17	6	8	3	10	7	5	2	9	6	6	2	3	1	3	3	4	4	0
C	6	1	2	3	1	2	4	4	8	0	2	2	4	3	1	0	2	1	0	1
D	5	0	7	6	12	5	7	8	7	5	4	2	6	6	9	2	1	1	1	3
E	1	0	4	1	3	2	1	0	0	0	0	0	2	2	2	1	0	0	0	2

There are many interesting results in the chart for the primary schools. For example, for every statement except 9 and M3, the vast majority of teachers had a firm opinion. Only in these two did one third of the teachers choose the no opinion option. And what are 9 and M3?

9. Educational Researchers are always contradicting each other's findings.

M.3. The work of Piaget, Dienes, and others has had a discernible influence on MY mathematics teaching.

This result for 9 might suggest that many of our teachers are unaware, as I suggested they should be, of the arguing between researchers. The result for M.3. needs to be viewed against the 50% who concurred with the statement and against the result for M.2.

The following statements received very high agreement percentages. (Very high means > 75%)

2. (90%). Some results of Educational Research which I have read have been absorbed into my teaching style.

4. (76%). The most useful form of Educational Research is school trialling of materials.

10. (85%). Classroom teachers carry out their own research day by day.

M.1. (73%). Maths is one area where I am aware of the value of research findings.

M.2. (75%). The work of Piaget, Dienes, and others has had a discernible influence on mathematics curriculum and teaching.

M.4. (73%). I need to know more about any results of Educational Research in the mathematics area which have been accepted as correct.

Also worthy of some note are:

8. (58%/32% disagree). Results of Educational Research are not communicated in an easily readable and understandable form.

11. (68%). I have more confidence in the advice of an experienced teacher than the findings of E.R.

12. (73%). I am more likely to apply my own experience than the findings of Educational Research when solving a problem in my classroom.

The following statement received very high disagreement percentages.

3. (74%). I have learnt very little from Educational Research.

5. (91%). Educational Research tells me nothing I didn't already know.

13. (78%). If a piece of Educational Research suggested marked changes in teaching practice, others may change but I probably wouldn't.

M.5. (68%). The best way to teach mathematics is with plenty of practice exercises and no research can improve that.

15. (64%). The main purpose of Educational Research seems to be to keep Educational Researchers in a job.

14. (62%). If E.R. had never begun, my teaching today would still be much the same.

Considering together, these results seem very positive to me. Educational Researchers have a good image among primary teachers. Educational Research results are heeded and used, but by an absorbtion/adaptation process, not by adoption. Teachers of mathematics (in primary schools) are asking to know more. Teachers are willing to consider change if the evidence is strong enough. (I base this last comment on the result of 13. But I do question its validity in my own mind because I wonder how many teachers would disagree with the statement and thereby place themselves in a "right wing" camp. Still, they could have opted for no opinion.)

It seems, in fact, that there is reasonable correlation between the teacher's view and the teachers' view. But the teacher would like to see more results from post-primary schools and overseas because he has a suspicion that general opinion would not be the same, because he detects a wider spread from radical to conservative among that group of teachers. Ant the teacher is aware that he has "cut up pigs" and is open to the same criticism that he levelled at others. But perhaps he is in a position to say to Educational Researchers "If you want to know more about the teachers' view of research, ask them."

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A COMMENT ON USING RESEARCH

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In this first session, Professor Romberg raised the question "why is it so difficult to use research in school practice?" Jacques Bergeron pointed out that there are discrepancies between a researcher's work and a teacher's work. Indeed, I think this is the main reason that we do not find immediate application of research in instructional practice. So, let me add a few remarks concerning the attitude of a researcher versus a teacher towards his or her own work, and the gap between both.

Research usually is planned long before carrying out research work. A researcher's honour is based upon his intention to take into consideration all aspects of the object of his research, or at least as many as possible. Also, if he cannot, he must make obvious why neglected aspects are not respected. His research work has to be repeatable and controllable,though in practice, certain circumstances do not allow to repeat the same work under equal conditions. A researcher at least has the claim to withdraw himself from the object of his work, not to make outcomes dependent on his individuality.

On the other hand, a teacher cannot plan his work long before doing it. He usually has to react quickly, to make decisions for the next day or even for the next few minutes, according to any situational conditions, which often change rapidly. In his practical work, he cannot take into consideration all components of the situation, not even reflect upon them. Instantaneous intuition is an attitude of a good teacher, the ability to manage unforeseen situations is an important skill in school practice. Furthermore, a classroom situation is unique and therefore, not to be repeated. There are autonomous individuals who cannot and must not be influenced totally. Finally, a teacher brings to the classroom his own individuality, his own person, as an unrenouncable component.

What consequences can we draw from the fact that there are so far-reaching differences about their work. First, we only can expect those research outcomes apt to be used in classroom situations which are not too specific, i.e., which refer to phenomena depending on not too many conditions to be checked in advance. Quick decisions do not allow to check a large number of alternatives before acting.

Second, I do not hesitate to suggest that researchers should be allowed to make generalizations from research outcomes even if this would not be appropriate from a scientific point of view. Researchers should dare to use generalizations of research outcomes in school practice not too scrupulously. In practice, we have opportunities to correct decisions. Within the domain of our question under consideration, we are facing decisions which do not have farreaching impact on what is actually going on in practice, but which may enrich practice.

Third, usually researchers do not use immediate implications of single research outcomes, interrelations between research outcomes and their possible applications in school practice are fairly complicated. Issues which can be used in instructional practice only can be derived from several research results, not from a single study. However, practitioners' actions do not have only one theoretical source.

Fourth, a research result will not have only one consequence to be drawn from it, rather several, and they need not be consistent with one another. Thus, the relation between research outcomes and instructional practice is often ambiguous. We should be conscious of these difficult and complex interrelations between theoretical knowledge and instructional practice in our discussions.

MEANS OF DISSEMINATION OF RESEARCH RESULTS

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Papua New Guinea

Papua New Guinea (PNG) is a developing country just to the north of Australia. It has a population of three and a half million Melanesian people. There are 720 different languages spoken within the country. PNG gained its independence from Australia in late 1975. Although by 1960 there was a skeleton primary school system across the country with a high percentage of expatriates staffing it, the secondary school system was virtually non-existent. Twenty-four years later, there is a fully integrated school system running from grade one through to two universities.

The system has had to withstand many pressures from within and without. With the continuing demand for progress, the schools have been seen as a major agent for change. Thus, many demands have been placed on them by politicians who do not always understand the limitations of schools. Within the system, there has been enormous growth. As well, immediately after independence, there was a determined push to have the system fully staffed by nationals. By 1978, the primary division had been localized. This emphasis has lessened in the past few years.

There are a great number of needs within the education system. Restricting the list to mathematics education, they range from the production of adequate text material for grades 1 to 12; upgrading of teachers' qualifications, a number in the Community Schools (years 1-6) still only have two years post grade six qualifications; adequate inservice training in all areas of mathematics and teaching. If that list looks daunting, when confronted by it in the real world, it looks even worse. In attempting to find solutions, appropriate reearch must be carried out. Part of the meaning of 'appropriate' will be research carried out in PNG. Too many supposed solutions have been proposed by visiting experts who have not had the time, or sometimes the will, to get out into the schools and become acquainted with the situation there. Some strategies which work in England or Australia may well be inappropriate in PNG because of language problems (Clarkson, 1983), different attitudes of pupils and teachers (Clarkson & Leder, 1984), difficulties in mounting inservice training, or an inability to supply schools with the ongoing, usable materials they would need to make an aids based program function (Roberts, 1981).

The research role of the Mathematics Education Centre (M.E.C.), and in some ways, the most crucial role it plays, is to attempt 'basic research projects, as opposed to curriculum development, which hopefully go some way to the finding of long term solutions to these needs. There is an unavoidable conflict of priorities in the system at present. Clearly at all levels there is a need for materials and mechanisms which will give stability to the syllabi that are taught by a teaching force which is comprised of young inexperienced nationals, and highly mobile expatriates. They want to know what is expected of them; at least what content is to be taught, and for the inexperienced teacher, either per se, or in PNG, what methods are most appropriate.

By comparison, in developed countries, even if there is no prescribed syllabi (for example, in Victoria, Australia) there is a tradition that suggests that particular topics are taught at specified times. Most teachers follow the tradition. The majority of textbooks are written with that understanding. In PNG there is no such tradition. Syllabi have been changed regularly. But such changes should be based on adequate research of the learning styles and teaching styles which are appropriate for PNG. There is some evidence that Western teaching styles may not be always appropriate (Clarkson, 1984). But to complete the vicious circle, there is not time for detailed research to be completed. Teachers need guidance and materials to use in their mathematics teaching now.

THE 'WHO' OF DISSEMINATION

Results of such research need to be disseminated to those who count. In Figure 1, some of the important people have been identified. It would seem important that if change is to be effected, then all members of the network should be informed first of the needs, and if possible, potential solutions. Each member has a contribution to make to the process of change in the classroom. Such coordinated dissemination is vital in a small system in which authority is centralized, but the schools are widely dispersed.

In Figure 1, the research unit has been placed by itself. This is imply to show the role of the M.E.C. Of course, there are times when curriculum developers or teacher trainers assume this research role. However, in PNG this is somewhat a rare occurrence unfortunately. The positioning of the various boxes in the figure also indicates the amount of time that the members of the various components are able to spend in schools. The one exception

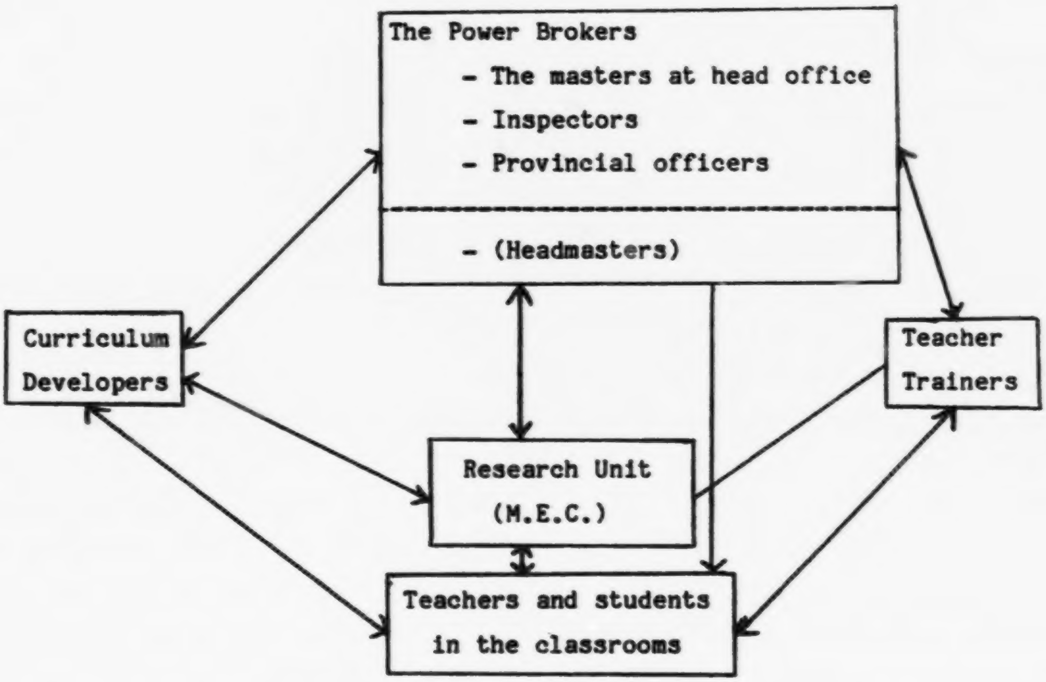
The Power Brokers
- The masters at head office
- Inspectors
- Provincial officers
- (Headmasters)
Curriculum Developers
Teacher Trainers
Research Unit (M.E.C.)
Teachers and students in the classrooms

Figure 1. The important members of the educational system in Papua New Guinea for mathematics education research dissemination.

is that of 'headmasters'. They obviously are in the schools, but a number rarely enter classrooms. There is a good excuse for many teacher trainers not to be in schools. They simply have an extremely full teaching load within their college. However, their being removed from the schools is still a fundamental criticism of the system.

'MEANS' OF DISSEMINATION

There are two basic ways used to communicate with the network depicted in Figure 1; by written word and verbally. The written word is necessary as a permanent record of what has been attempted and the avenues explored. This is vital for those who will hopefully build on the present work. As well it is a contact with members of the network who are rarely if even met personally. Thus, the M.E.C. has two series of reports which are issued on an irrelgular basis for free; the Mathematics Education Centre Reports are aimed at a wide general audience, and the Mathematics Education Centre Technical Reports are sent out to specific groups. It is gratifying to note that some of the research reported thus has been built on by others. Some of the reports have also been incorporated into courses undertaken by teacher trainees; an opportunity for future teachers to study their own education system. Probably, however, the more meaningful reporting of research occurs by word of mouth when qualifications and implications can be spelt out.

To this end, the M.E.C. has hosted a conference on mathematics education for the last four years, not only to report on our own research, but to give opportunity for comment and discussion on other mathematics educational matters pertinent to the fairly broad group which has gathered each time. Of course, one conference per year is not enough. However, there are other opportunities to discuss our research during inservice work, with comments made during the various Syllabus Advisory Committee meetings of the Ministry of Education, and so on. Perhaps the most important method remains the personal contact with key workers in the Curriculum Unit of the Ministry of Education, with staff at the University of Papua New Guinea and in the Teachers' Colleges, with the curriculum advisors and inspectors at the provincial level, and the inservice work carried out for teachers who are at the chalk face. Such contacts are vital if the research ideas are to flow on quickly.

'WHAT' TO DISSEMINATE?

Once the target populations have been identified and means of dissemination have been worked out, what do you tell them? It is relatively easy to write up an academic report giving background, method, results, and the final discussion noting that the results will need replication. Of course, that is important. It should be done. But it should not be the common procedure to leave a report on research at an academic level in Papua New Guinea. The implications for curriculum development, teacher training, refining the system or, getting down to bed rock, teaching in the classroom should be drawn out. Different implications will be emphasized depending on the audience which is being addressed.

An example of this can be taken from the area of error analysis. Some use has been made of a procedure first devised by Ann Newman in Victoria (Clarkson, 1983; Clement, 1980; Newman, 1977). The procedure was devised to investigate students' problems with written mathematical problems. Table 1 gives the six basic questions which are used by the investigator and the corresponding six error categories. The message for the curriculum developers may be the list of words, symbols and phrases which students find difficulty. Teacher

Table 1

Category of Errors Developed by Newman With Key Questions and Statements

Reading	Read the question to me.
Comprehension	What is the question asking you to do?
Transformation	How will you do the question?
Process skill	Complete the question for me.
Encoding	Write down the answer.
Careless	

trainees should learn that it is not just in the 'process skill' area that students have difficulty. The power brokers can be made aware that words are important in learning mathematics, and students should be expected to use them both in verbal and written forms. Thus, headmasters and inspectors should expect to see students talking about their work in class. And the teachers? They appear to be interested in all of the above.

But leaving it just there, a stark reporting of the results does not do justice to the research. The results should be amplified by why the researcher feels they are important, why was the investigation carried out in the first place, what methods were used, and so on. In reporting research in this way, the audience starts to feel part of the wider scene. In fact, it then is a small step to incorporating the audience into the research process, if they are willing. The Newman technique is an ideal vehicle for this where interview situations can be easily set up for teachers and teacher trainees with very little training involved.

Perhaps the final answer to 'what to disseminate' is the ethos of the research, and the hope that the audience becomes infected by it. It is my belief that teachers learn far more from research if they are part of it, than if they are mere providers of children to be researched on.

There is a need for one qualifying question to be asked of the above. How far should the implications of research bedrawn out by a researcher who is not part of the system, and who may not fully appreciate all the intricacies of the system in which the development must take place? It is hoped that comments by outsiders will bring a useful extra perspective to bear on problems that members of the system are grappling with. However, it is an issue which we try to keep in the forefront of our thinking. I guess our main rule of thumb in this is to draw out fairly broad implications in our written reports, and leave the detail for face-to-face discussion.

CONCLUSION

For research to be effective, it must reach the classroom. There is the direct route by contact with teachers, or more indirect ways by influencing curriculum developers, teacher trainers and the power brokers of the system. Once the audience is identified, specific means appropriate to each member must be employed. Finally, it is not just results which should be conveyed, but the whole ethos of the project. For this to happen, all members of the network must keep talking and listening to each other.

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COMMUNICATION OF RESEARCH RESULTS IN AUSTRALIA

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Since the advent of the Mathematics Education Research Group of Australia (MERGA) in 1977, there has been a rapid growth in research in mathematics education in Australia. Contrary to views sometimes expressed by teachers and academics, there is much in this research which is relevant to teachers. There is a need, however, for the research to be communicated more effectively: it is quite unrealistic to expect teachers to read theses or journal articles to discover what is under investigation.

In discussing the communication of research, I have chosen to consider four questions which to me are the most interesting, and/or most in need of attention. These are: who does the research? where does it appear? who uses it? who should use it?

The first question is not greatly difficult to answer and perhaps could be left untouched. However, I want to discuss it briefly since I think it affects the other questions. It may be best though to begin with the question of who the research should be read by. There is sometimes an assumption that the intended audience for reports of educational research are teachers. It is probably true of much research that if the findings are important, then they must be important to teachers, but there are other groups to whom such reports might also be communicated. For example, some of the research findings regarding different levels of interaction of male and female students with their teachers might achieve most if we communicated them to the students! Other groups for whom research findings may provide direction are administrators, textbook authors, curriculum planners, lecturers in education, and students of education. having said that, the main group for whom research results have significance are the teachers.

I now return to the first question: who does the research? In most cases, the answer is: students working towards a higher degree. For example, of the studies listed by Blane (1984) 103 have been conducted, or are being conducted, in order to produce a thesis at the Maters or Doctorate level. Certainly there are a number of studies not having a thesis as their primr purpose - for example, Blane (op.cit) lists 40 projects which have been funded by one of a number of interested parties. Nevertheless, the majority of research is conducted for the purpose of gaining a higher qualification and it would seem that this is also the case in the USA (cf. Kilpatrick, 1981). It follows then that a primary source of research material will be theses produced by students. "Unpublished doctoral dissertation" is a very common bibliographic listing and it must be admitted that most dissertations deserve to remain unpublished. This is not necessarily a criticism of what is contained in theses, but simply a recognition that they are produced to satisfy a number of criteria of which readability and conciseness are but two, and usually not paramount. The important consequence of this is that in cases where findings of educational significance have been made in the pursuit of a higher degree, the author must publish these results in some more public form, else they may never reach the majority of teachers.

Studies not directed towards a thesis are frequently of the kind referred to earlier, that is those arising from funded research and these are perhaps more likely to address questions of direct classroom significance. The primary vehicle for publishing these is the report issued at the end of the project, along with any interim reports produced along the way. Again, it is ane xception when such a report becomes a document read by a wide audience. Thus, there is once more the need to publish findings where teachers are more likely to read them.

After theses and research reports the next most likely outlets for details of research are journal articles and conference proceedings. In this respect, May 1977 was a particularly significant date for mathematics education in Australia for it was then that the first meeting of MERGA took place. This brought together a wide body of professionals with an interest in research in this area. Thirty-four papers were presented and it was very clear to all present that the organizers of MERGA had correctly perceived that a considerable body of research in mathematics education had built up with no suitable outlet. (Jones [1979, 1983] has given a more detailed account of MERGA's role in this regard.)

The reasons that this was so are worth examining briefly. The first was that mathematics education was very much the Cinderella of education studies. Until the sudden expansion of the College of Advanced Eduction system in the late 60s and early 70s, there was very little research in mathematics education in this country. Over a short period of time, staff who had been engaged almost exclusively in educating trainee mathematics teachers found that their roles had altered: they were now required to engage in educational research. Many also felt a need to upgrade their qualifications and for a sizeable proportion of the group, this was accomplished through a period of study overseas. Having returned, they were in a position to report their research to the newly emerged mathematics education community and it is, therefore, not surprising to note that many papers presented at MERGA I reported on research carried out overseas in the gaining of a qualification.

Before the formation of MERGA, there were, of course, other vehicles for publication. For example, Collis (1971, 1973) published detailes of his research in The Australian Journal of Psychology and one finds occasional articles relating to mathematics education in The Australian Mathematics Teacher (for example, McQualter, 1974; BRinkworth, 1977). Generally, though, the latter journal concerned itself much more with mathematics per se, rather than mathematics education, and the former would not have been perceived as the suitable place to publish most of the papers presented at MERGA I. Thus, the success of MERGA can be attributed to its providing a suitable outlet for articles on mathematics education just when a pool of recent research has built up. Before 1977, there had been only a trickle of such articles in Australia, but even since, there has been a considerable stream. Most of these have appeared in MERGA publications, but Conroy (1983) located twenty-six articles appearing in other Australian journals since the inception of MERGA.

Since 1977, two developments have occurred. The first has been an increase in the number of Australian studies. In 1977, fourteen papers reported on research studies (as distinct from papers discussing adminstrative or pedagogical issues in a more general way) and of these, eight had been conducted outside this country. By contrast, in 1982, there were twenty-one studies reported of which eighteen were conducted in Australia.

Second, there has developed a demand for a publication of higher status than conference proceedings. In recent years, a number of Australian mathematics educators have tended to publish their more scholarly articles in overseas journals such as Educational Studies in Mathematics and For the Learning of Mathematics. At the annual general meeting of MERGA in May 1982, it was agreed that the time had arrived for this body to produce its own refereed journal. Two numbers of Research in Mathematics Education in Australia have now appeared and clearly, it is attracting the kind of articles previously published elsewhere. Researchers will still see it as appropriate to report their work in overseas as well as Australian publications but now the local articles should achieve comparable status.

I would like to return now to the comment I made at the beginning of this paper. It is true that when we think about the communication of research, we naturally think about journals and conference proceedings. But these have their shortcomings. They are invaluable to those involved in research since an early step is to examine the literature, and without the regular publication of research reports, the task of the researcher would be much more difficult. But the question must be asked: has this process produced a small incestuous community of literature in which the only people who read and refer to the results of a research project are those who will use it to produce another? To suggest that this is always the case would be unduly cynical. To suggest that it is never the case is naive.

Returning now to other modes of communication, I would like to argue that oral communication is probably the most effective. One form of oral communication is the lecturer-student one. For example, those of us who are involved in teacher education have almost certainly made our students aware of Marilyn Suydam's (1976) review of research on the use of calculators in classrooms. But generally at the preservice stage, students are not particularly receptive to research reports, they are concerned with more pratical matters.

Finally, there is the view sometimes expressed that research is irrelevant to teachers (Pateman, 1982; Kilpatrick, 1981) but I am not so convinced by this argument. It may be true as Kilpatrick suggests that teachers would benefit more if they were involved in the research, but realistically in Australia, this could involve far too few teachers. Not only would far-flung country schools miss out, but many suburban ones as well.

Inservice education, it seems to me, is the ideal period in which research results can be communicated effectively. At this level, the discussion of research reports can have quite an impact. The real problem with this approach is not its effectiveness with the recipients, but the small percentage of the teacher population who are exposed to such courses. With one or two notable exceptions, higher degree courses in mathematics education have attracted few students. Certainly only a small percentage of the teaching body could be involved in this way.

Perhaps we need to find some other way to expose teachers, at the inservice level, to this kind of approach. Not a large percentage undertake formal post-graduate study but many do participate in other kinds of inservice programs. These could usefully have a more substantial theoretical component than preservice courses since it is generally acknowledge that theory is more meaningful after a period of teaching experience. We might note here that Cockcroft (1982, p. 228) asserts the importance of research in the inservice support needed for teachers.

I am prepared to argue that research in mathematics education is not irrelevant to teachers, but I will concede that the research could be much more closely linked to practice, as advocated by Milton (1983). If we expect short-term solutions to classroom problems then of course the research will not be seen as relevant.

If, for example, we consider the research questions which were discussed at the 1982 MERGA conference, their importance to teachers is undeniable. But what must be recognized is that their value may not be restricted to the results. Simply making teachers aware that a particular research question is valid may be the most significant outcome. The fact that a question is worth asking can be a challenge to digmatically held views. If as an outcome we produce a more reflective attitude, then there must be long-term benefits of a more subtle kind than might result from some clear-cut findings to narrowly framed questions. It can be argued that the only questions which will have clear cut answers are very narrow ones. The more profound questions, especially those in the area of curriculum, will be much more difficult to answer and this will require a cooperative effort from teachers, researchers, and many others.

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COMMUNICATING RESEARCH IN MATHEMATICS EDUCATION TO SCHOOL PRACTITIONERS[1]

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One can point to very few examples of research studies having a direct and lasting effect on curricular or instructional practices in mathematics education. Yet, research can have a substantial influence on educational practice in more subtle ways. But, in order to have any effect on practice, research needs to be communicated effectively to a large audience of educational practitioners, who have different interests, needs, and perspectives.

COMMUNICATION: TO WHOM?

Research results speak in different ways to different groups of educators. Teachers, administrators, curriculum developers and supervisors, and teacher trainers each look to research with somewhat different needs, interests, and perspectives. Although each practicing educator has his/her own unique perspective, one might make the following generalizations.

Teachers are interested in techniques that work with students. Because they are in the "day-to-day trenches" and generally take the credit or blame for what students learn or do not learn, many teachers are reluctant to try new ideas or methods unless they have been "validated" by inclusion in tests or textbooks or by the mandate of an administrator. Yet, because techers are in daily contact with students, they are the key persons researchers should try to reach if research findings are to be incorporated into the classrooms.

Adminstrators generally are looking for activities that will provide evidence to parents (and the world) of successful academic performance of the students. Improved and/or high test scores are the usual indicators of success that administrators respond to.

If they have a background in matheamtics and/or mathematics education, curriculum developers/supervisors and teacher trainers are more likely to be philosophically akin to researchers in mathematics education. Generally,

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they are interested in research that supports instructional practices that they believe should occur (but often do not) in the classroom. They frequently can be the link between researchers and both teachers and administrators.

COMMUNICATION: OF WHAT?

When research is examined for its educational implications, it is important that all aspects of the research be considered. The most obvious aspect of research to examine is the results. Teachers and administrators will, of course, be interested in research that finds strong positive change in student performance. But there are many reasons why results should not be the only (or perhaps not even the major) aspect of research that is considered. For example, statistical analyses are often flawed and the flaws are difficult for the untrained reader to detect; reports of "no significant difference" may be as important (but more difficult to find published in journals) as those reporting differences; and some of the best research done in recent years has used more qualitative techniques (e.g., case studies) to report findings.

Everyone is interested in the results of an educational research study, but there are other aspects of research that should also be considered for their significance and implications for practice. The underlying questions and theoretical constructs of a research project can be significant. Teachers may share the same concerns as the researcher and come empirically to the same conclusions, but they may not have thought critically about the underlying constructs that relate to the concerns. Educators may wish to examine why a researcher was interested in a particular project, what areas it relates to, or how the results might be interpreted.

The third aspeact of research that should be of interest is the nature of the tasks used in the research. Educators may be able to use the tasks directly in the classroom. Research tasks can often be effectively adopted for instructional use or for the purposes of evaluation.

COMMUNICATION: HOW?

How can research be effectively communicated to school practitioners? In this section of the paper, we discuss five different vehicles for such communication: published reports, professional organizations and meetings, inservice education, preservice education, and special thematic conferences.

Published Reports. Written interpretive reports in journals, books, or other published materials, which are directed at those working in schools, can be an effective way to communicate research to practitioners. There are a number of journals, such as the Mathematics Teacher and the Arithmetic Teacher, published by the National Council of Teachers of Mathematics; Educational Leadership published by the Association for Supervision and Curriculum Development, and the Phi Delta Kappan that are widely read by school practitioners. These journals provide a variety of articles, including research summaries, on current issues in education.

Reports of single research studies, because of their relatively narrow focus, are often not of immediate interest to school practitioners. However, reports of sets of related studies or summaries of research, written especially for school practitioners, can be very effective in communicating research findings. Mathematics Education Research: Implications for the 80s (Fennema, 1981), Research Within Reach (Driscoll, 1981, 1982), and Classroom Ideas from Research on Secondary School Mathematics (Dessart & Suydam, 1983) are examples of books containing research summaries that convey research findings by describing classroom situations very much like those encountered by most teachers. Briefer research summaries written for practitioners also appear as journal articles or chapters in books. For example, in the area of problem solving, the chapter by Suydam (1980) and the recent article by Silver and Thompson (1984) were both written for an audience of school practitioners. These descriptions, along with a discussion of the methods and findings of the researchers, can be the starting point for teachers to question the teaching practices that routinely occur or to focus differently in learning difficulties they observe. They may also inspire teachers to question their own students to conduct a mini-research project in the classroom to see if their students respond in the same way as reported in the research summary.

Interpretive testing reports, such as those issued by the National Assessment of Educational Progress (NAEP, 1983) and the California Assessment Program (CAP, 1983) can help educators examine their instructional program. Interpretive reports fo NAEP results are also published in journals written for teachers, such as the Arithmetic Teacher (Carpenter et al., 1983), the Mathematics Teacher (Lindquist et al., 1983), and the Elementary School Journal (Carpenter et al., 1984). Both NAEP and CAP have items in their mathematics tests that attempt to assess problem solving, as well as traditional, lower level skills.

The CAP tests all students in grades 3, 6, 8, and 12 in California Public Schools. Reports to each school provide detailed information on how well the school is performing in comparison to other California schools in more than thirty different mathematics skill areas. Since, in some grade levels, approximately fifty percent of the test items assess problem-solving processes or matheamtical applications, the test results can provide not only a rich source of information about student achievement in this important area, but also an impetus for schools to examine the relevance of their educational program to the problem-solving objectives promoted by the test (and supported by the state of California). This has become a substantial way for research to influece practice, since the results of student performace can lead directly to curricular or instructional modifications in a school's program. It is also important to note that all objectives and items for the CAP were written by a committee of California mathematics educators, with practitioners and researchers represented on the committee, so that the influence of previous research can be seen in the development of the test.

Professional Organizations and Meetings. Professional organizations of mathematics educators (local, state, national, and international) can provide the environment for mathematics educators from all levels to come together, get to know each other, and work to promote common goals. In San Diego, there is a very active organization, the Great San Diego Mathematics Council. The membership includes teachers of preschool through university levels. Because of the large number of officers and committees, mathematics educators throughout the area have developed truly colleagial relationships. This promotes the sharing of expertise and experiences between educators and researchers.

Meetings and conferences of local, state, and national organizations of mathematics teachers or supervisors are a way of communicating research to school practitioners beyond those actively involved in the organization. Although in California, at our state and local meetings, we have very few sections identified as "research," researchers are frequent presenters. Their presentations may focus on aspects of their research or its implications that are particularly pertinent to practitioners. There are special research-related sections at the NCTM annual meeting, and they attract as many school practitioners as researchers. There has also been a growing relationship between the National Council of Supervisors of Mathematics (NCSM) and the research community.

NCSM has sponsored research activities for its members in conjunction with its annual meeting.

Inservice Education. Inservice education activities for school practitioners takes many forms from short, after-school workshops to summer institutes of several weeks' duration. Inservice education is often initiated at the school cite or district level. In some instances, the state or university may be the initiator.

Inservice activities planned by school or district personnel can effectively incorporate research into the activities if those responsible for the planning are aware of the need and value for doing so. Informed mathematics supervisors or teachers can serve as resources to their school or district to see that inservice activirties reflect the implications of research findings. They can be an effective link between research and practice.

When public interest in mathematics education is high, as it is now, state education agencies and universities are frequently able to obtain funding and offer special inservice education programs. For the past two years, the California Mathematics Project has offered special inservice programs at approximately ten sites each year throughout the state. This program is funded by the state and administered through the University of California/California State University system. Although each site plans its own project, many of the sites have brought researchers and practicing mathematics educators together as instructors for these summer institutes for mathematics teachers. (Grade levels of teachers vary at each institute. Some projects are for teachers of grades K-12, others have a more narrow range. For example, the San Diego Mathematics Project was for teachers of grades 7-12.) Research findings have been woven into the instructional program at these institutes so that teachers can see the implications of research for their classroom. Researchers have been invited to make presentations to participants at many of the projects. Presentations are often followed by discussion of common concerns.

Preservice Education. This is the area for which we can point to the fewest examples. Little systematic effort is made to incorporate the communication of research to future teachers. Most of the professional preparation that preservice teacher candidates receive is very general and does not focus on specific subject matter. Some books that are used for mathematics content or methods courses for prospective elementary school teachers do include some ideas or findings from research. At the secondary level, it would be very unusual for a text for a content course to contain any information about research related to the learning of that topic or related topics. We suspect that the primary responsibility in this setting rests on the shoulders of the professor who teaches the course. Whether or not research gets communicated to future practitioners seems o depend greatly on the knowledge and initiative of individual course instructions.

Special Thematic Conferences. When researchers come together for small interest meetings, they might consider the possibility of holding a special conference for school practitioners in the locale. A conference can be sponsored jointly by a local education agency and the university hosting the special interest meeting. An example of such a conference was held in June 1983 in San Diego. Thirty-six researchers of mathematical problem solving attended a three-day meeting at San Diego State University. The meeting was funded by a National Science Foundation grant and was organized by Edward A. Silver. On the day following this meeting, approximately 200 school practitioners attended a conference entitled "Teaching Problem Solving: Research Can Make a Difference," at which 13 of the researchers made presentations. The latter conference was sponsored jointly by San Diego State University and the San Diego County Office of Education. Comments from those who participated--the presenters and the audience--were extremely positive and indicated a strong interest in participating in other conferences similar to this one.

COMMUNICATION: WHAT ELSE?

All models of research communication discussed thus far have assumed that the flow is from the researcher to the school practitioner. This is the customary model accepted by both researchers and school practitioners. However, the term "communication" implies a two-way channel. If research is going to be relevant to classroom teachers and school administrators, it must address practical classroom concerns. If researchers want practitioners to consider the imlications of their research studies, then they must be willing to listen to the questions that are important to school practitioners.

One natural outgrowth of improved communication between teachers and researchers might be more teacher involvement in research. As teachers develop questions concerning mathematics instruction in their classrooms, they can devise, with or without the assistance of an experienced researcher, mini-research projects to conduct with their students. The results of such research could then be tested on a larger scale. When teachers seem themselves as both instructors and researchers, effective two-way communication between researchers and school practitioners can be realized.

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DEVELOPING TEACHERS' STYLES: A REPORT ON THE DISSEMINATION OF TWO RESEARCH AND DEVELOPMENT PROJECTS "DIAGNOSTIC TEACHING" AND "TESTING STRATEGIC SKILLS"

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It is often said that children learn more from what they do, than from what they are told. Indeed, this has been a philosophy underlying the movement towards a more process oriented curriculum, advocated by so many mathematics educators. In this paper, however, I would like to apply this philosophy to teachers. Governmental reports, professional associations, and other educational bodies are continually exhorting teachers to change their style of working. Their arguments may be supported with research findings and statistics, but they still seem to have only a minimal imnpact on the education that our children receive. In the confines of the classroom. HMI reports show that many teachers continue to operate in much the same way as they have always done - a period of exposition by the teacher followed by closely guided imitative exercises for the pupils. Others appear to leave the responsibility for learning to the textbook or 'individualized' scheme, which often atomise and dehumanise the subject, giving the impression that mathematics is a miscellaneous collection of arbitrary rules, techniques and tricks. David Wheeler pointed out, in his plenary paper to the PME conference in Israel last year, that the "single problem most urgent and important for us to solve" is that even "our best research efforts have no discernable effect on the education that our children receive" (Wheeler, 1983).

In discussions at the Shell Centre, we often distinguish between four levels of research and curriculum development work, all of which we feel are essential and complement each other:

L	Goals for Learning -	what can we reasonably expect pupils to achieve?
T_1	Teaching Possibilities -	can we devise teaching methods that enable our pupils to achieve these goals?
T_2	Realistic Teaching -	can we develop teaching methods that are generally accessible to teachers?
C	Curriculum Change -	how can we implement this system on a large scale?

In the past, serious work everywhere, including our own, has tended to concentrate on the first two levels (L and T_1). There is, however, an enormous gap between T_1 ("there exists a teacher, usually in the development group, who can use these methods effectively") and T_2 ("what will the second worst teacher in your department make of this material?"). This may partly explain why so much excellent work has failed to be taken up more widely.

My own view is that if research is to have any impact at 'C' level, it must provide teachers with sufficient methods and resources to enable them to discuss and assess the effectiveness of their own teaching. Somehow, they must become participants in the research process, not merely the receivers of reports written by 'experts'. In addition, developmental work must also take account of the pressures that teachers are under from overcrowded and often inappropriate syllabuses, from the examination system, from a lack of physical resources, from large and often difficult classes, and so on. Such pressures exert a narrowing effect on the content and style in which mathematics is presented, and make any large scale developments very difficult. For success at 'C' level, any innovation should, therefore

- make the teacher's job easier,
- make it more fun,
- tackle a problem they know they have got,
- have some outside pressure behind it.

Many reforms fail on all four counts; they are not easy to meet, but should not be ignored if one is serious about 'C' level.

In this paper, I shall discuss both a models 'T_2' attempt to involve teachers in carrying out their own research and development by means of a short, experimental, inservice course, and a more ambitious 'C' level attempt, involving a national examination board, to encourage a more balanced range of classroom activities, particularly those highlighted by the Cockcroft Report (1982): problem solving, practical mathematics, discussion and open investigation. I am, therefore, not merely concerned with the dissemination of research results, but rather with encouraging teachers to adopt a learner-centred rather than subject-centred approach and raise their awareness of how it feels to struggle to learn and do mathematics.

THE DIAGNOSTIC TEACHING PROJECT

Background. The Diagnostic Teaching Project arises from the now extensive studies of mathematical understanding (e.g. Hart [1981], APU [1980, 1981]) and involves the development of task in a number of topics which expose pupils' misconceptions in a manner inspired by Piaget. This method recognizes the conceptual systems in which unsuccessful pupils operate and designs tasks in which the use of an inadequate conceptual scheme will lead to a 'conflict' between contradictory results. Pupils are thus encouraged to seek a resolution, which will hopefully involve restructuring their conceptual systems. This is essentially a reflective activity, and in practice, it necessitates a great deal of classroom discussion. Correct concepts and methods are then consolidated using exercises with a built-in feedback of correctness for the pupil. This approach challenges some traditional teaching notions - in particular, that it is the important to define a concept fully and correctly at its first introduction. We find it rather surprising that this view can still be maintained, since it is very clear that misconceptions and partial conceptions are part of a child's normal course of development, and these can only be changed if they are brought into awareness and subjected to conflict and correct notions. Many teachers, while accepting this, are still reluctant to deliberately expose and discuss mistakes in the classroom, and yet this is the essence of teaching by cognitive conflict and has been shown to be highly successful in a number of experiments (see Swan, 1983 for example).

Eventually, we hope that an outcome of the project will be a collection of illustrative 'packages' of material for teachers on different topics, each comprised of three elements:

1. Diagnostic Tests designed to expose and classify common errors and misconceptions, together with a general description of the conceptual field, illustrated with videotapes of pupils.
2. Lesson Sequences containing worksheets, discussion material and teaching notes.
3. Design Principles which enable teachers to develop their own lessons based on a diagnostic philosophy in other areas of the curriculum.

Such a package concerning 'The Meaning and Use of Decimals' (Swan, 1983) is nearing completion and is now available for teachers to use.

A short, experimental, inservice course. Recently, we conducted a short inservice course for teachers of pupils aged 10-16 which was intended to

- acquaint course members with the recent research on children's understanding of mathematics and on the design of more effective teaching methods.

- enable members to conduct experiments of their own to test new teaching approaches.
- assess the reaction of some teachers to our methods and materials.

In this venture, we were aided by our colleagues from the "Strategies and Errors in Secondary Mathematics" (SESM) project, Kathleen Hart and Lesley Booth, who contributed their research results and teaching material on the topics of Ratio and Elementary Algebra, respectively. (Now published in Hart, 1984 and Booth, 1984.) Our own input concerned the topics of Decimals (see Swan, 1983) and Directed Numbers (see Bell, 1982, 1983).

Nineteen teachers attended the course which was introduced by a two-day conference in July 1983, and followed up by five one-day meetings during September 1983 - April 1984. Three topics were introduced at the July conference, and the fourth, Ratio, at the second one-day meeting in January. In each case, the topic was introduced by two one and a half hour sessions, the first giving an outline of the common misconceptions encountered by pupils, and the second discussing teaching ideas and materials. Small working groups were formed, in which they planned and discussed teaching experiments. Each member was encouraged to perform two experiments in different topics during the course, which were usually of the pretest/treatment/posttest (and sometimes delayed posttest) kind. In some cases, teachers adapted and invented ideas for materials based on diagnostic principles, while in others, they used ours exactly as presented.

During the one-day meetings, several research reports were presented to the participants concerning specific aspects of teaching material: using diagrams estimating and checking, using games, and tasks which reverse the usual classroom roles, for example, where children are invited to invent questions or mark homework and diagnose errors made by others. Many of these ideas were subsequently used by members in their own classrooms, and theseexperiences were shared and discussed later in the course.

A Brief Evaluation of the Course. Throughout the duration of the course, members were invited to give us (anonymous) feedback on their reactions to it. The following remarks are based upon this feedback.

1. Overall, their reactions were very favourable. Members had attended the course for a variety of reasons ("to improve my teaching"; "to keep abreast with research"; "to meet other people and exchange ideas"; "to find out

why children have problems"; "to acquaint myself with new teaching material") and they all felt that their needs had been met very well by the course.

2. The main criticism appeared to be that we rather overwhelmed course members with research results at the beginning, during the two-day introduction. As time progressed, however, the input by the course leaders declined as members became more involved in sharing experiences from their own experiments. Overall, most felt that the balance between discussion and practical work was about right. To quote one member: "Initially, just another set of academics telling how to teach - I quickly realized the value of the conflict approach and the research that had been done." This supports the view that if teachers are to accommodate new styles into their teaching, then they do need to appreciate their value from first-hand experience.
3. The experiments conducted by members were very varied, but nearly everyone obtained at least one set of encouraging results. For example, seven experiments were conducted on Decimal Place Value, and everyone found considerable gains in the Pre-Posttest results. The three participants who administered Delayed Posttests were all surprised to find that their pupils had continued to improve, without further teaching. As most experiments were conducted informally and the analyses lacked rigour, few conclusions can be drawn from the actual data generated. However, this was not the intended outcome. More significant was the involvement and depth of discussion generated by teachers participating in their own research. This also helped them to assess and appreciate experiments conducted by others.
4. At the end of the couse, we asked members if there had been any modification in their attitudes towards teaching mathematics. Here are some fairly representative replies:

   "Yes. Much more concerned with mistakes children make, rather than looking for correct answers."

   "I think more about teaching material. Does it help to bring difficulties to light and eradicate them?"

   "It has assisted me in changing staff attitudes towards pupils' mistakes."

In conclusion, we were greatly encouraged by the way in which this course appeared to develop the awareness of teachers to the nature and extent of pupils' misconceptions and increase the range and effectiveness of their own teaching styles. However, we are well aware that these were not 'typical' teachers (most do not attend courses!) and that if we are to make a more widespread impact, then somehow teachers must be given the resources and time to get together, independently, and reflect on their teaching within their own schools. The proposed packages of material may provide resources for this, but, of course, there is, as yet, no way of ensuring that teachers will feel the need to consult such packages. The "Testing Strategic Skills" project, outlined below, attempts to overcome this problem by involving the lever of a large public examination board.

THE TESTING STRATEGIC SKILLS PROJECTS

Background. As stated earlier, the aim of this project is gradually to introduce into the classroom a more balanced range of activities, particularly those highlighted by the Cockcroft Report: problem solving, practical mathematics, discussion and open investigation. It explicitly recognizes the fact that public examination boards effectively determine the school curriculum by the syllabuses they set and this in turn has a direct influence on the nature of the mathematics published in textbooks and taught in classrooms. Even though many teachers have considered the inclusion of more problem solving and investigational work in their lessons, most do not because exposition and the consolidation and practice of routines are more appropriate when preparing pupils for the stereotyped, 'standard' problems set in examinations. The crowded nature of the curriculum also reduces the time available for discussion and discovery. Alternative, more radical forms of examination have been designed in the past, but these necessitated great style shifts in teachers, and it is, therefore, not surprising that they have had only a limited appeal. People rarely switch to risky alternatives when they are comfortable in what they are doing.

We, therefore, decided to try to introduce into existing examinations, new kinds of questions which we hope will encourage the 'missing' activities described above. These questions will be introduced gradually, and with the support that will be necessary if most teachers are to adopt the changes in content, attitude, and above all, teaching style and strategies that are implied. (This support does involve a reduction in syllabus content.) The project is a joint enterprise of the Shell Centre and the Country's largest public examinations board, the Joint Matriculation Board, which services about one-third of the secondary schools, particularly in the north of England.

The approach is gradual for two reasons. First, teachers can only reasonably be expected to absorb new elements in small quantities, and so an approach which leaves all but a small proportion of the curriculum untouched helps to sustain and build confidence. Secondly, the development is a slow and demanding process and it does not seem possible to produce materials of the quality required over the whole curriculum at once.

The teaching modules developed so far each correspond to roughly 5% of the two year examination course, which amounts to three or four weeks teaching, and to one question of the examination. Each module consists of three elements:

specimen examination questions, with sample answers (not model answers) and marking schemes, and an accompanying explanation of the scope of the module,

classroom materials - offering detailed teaching suggestions and pupil worksheets,

support materials - to provide ways in which teachers, either individually or in collaboration with colleagues, can develop their teaching styles and explore the wider implications of each module. These materials, which provide the basis for a short 'do-it-yourself' inservice course, include the use of video and microcomputer software resources.

These modules are being carefully developed by groups of teachers working with the Shell Centre, with structured classroom observations in a sample of schools representative of those who take the Board's examinations. The first line of development has been in the O-level examination, and is thus aimed at the top quarter of the ability range at 16+. The development process has been worked out in detail in this context. The first module Problems with Patterns and Numbers, is now available; the first question on it will be set in the JMB 1986 O-level Mathematics Examination. The second module, on the Language of Functions and Graphs, is now at the stage of pilot trials in classrooms. Two further modules are anticipated and will be concerned with applications of mathematics to Consumer Decisions and Everyday Problems.

An Outline of the First Module - "Problems with Patterns and Numbers. To illustrate how these modules can help teachers to develop their style, we will give a brief outline of the first module, together with a few illustrations. 'Problems with Patterns and Numbers' aims to develop the performance of children in tackling mathematical problems of a more varied, more open and less standardized kind than is normal on present examination papers. It emphasizes a number of specific strategies which help such problem solving. These include the following:

- try some simple cases
- spot patterns
- check regularly
- organize systematically
- find a general rule
- explain why it works

Such skills involve bringing into the classroom a rather different balance of classroom activities than is appropriate when teaching specific mathematical techniques; for the pupils, more independent work and more discussion in pairs or groups, or by the whole class; for the teachers, less emphasis on detailed explanation and on knowing the answers, and more on encouragement and strategic guidance.

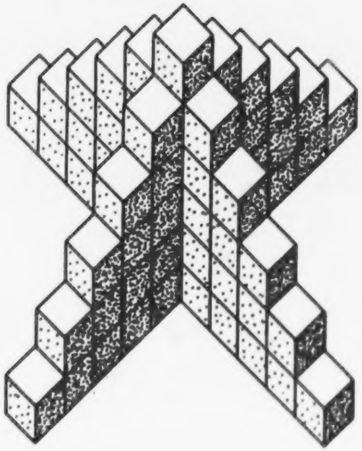
Below we give just two examples of specimen examination questions taken from the Module. The marking schemes are designed to give credit for the effective display of strategic skills, in particular for:

- showing an understanding of the problem.
- organizing information systematically.
- describing and explaining the methods used and the results obtained.
- formulating a generalization or rule, in words or algebraically.

Full marking schemes, illustrated with actual pupil answers are given in the Module book.

The classroom materials offer resources by which pupils can be prepared for the questions on the examination. They are organized into three Unites (A, B, and C, each of which is intended to support roughly one week's work), together with a problem collection providing supplementary material for the quicker student, or for revision. Through the three Units, the guidance provided to the pupils is gradually decreased so that by the end, they are facing challenges similar to those presented by the examination questions. Unit A consists of a series of worksheets based around a set of problems, which aim to teach a number of powerful problem-solving strategies, and demonstrate their "pay off". Unit B gives the pupil less guidance, now in the form of "checklists"

SKELETON TOWER



(i) How many cubes are needed to build this tower?

(ii) How many cubes are needed to build a tower like this, but 12 cubes high?

(iii) Explain how you worked out your answer to part (ii).

(iv) How would you calculate the number of cubes needed for a tower *n* cubes high?

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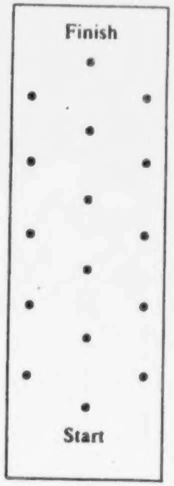
THE CLIMBING GAME

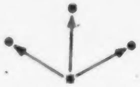


This game is for two players.

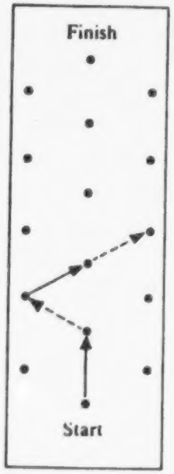
A counter is placed on the dot labelled "start" and the players take it in turns to slide this counter up the dotted grid according to the following rules:

At each turn, the counter can only be moved to an *adjacent* dot *higher* than its current position.

Each movement can therefore only take place in one of three directions:



The first player to slide the counter to the point labelled "finish" wins the game.



(i) This diagram shows the start of one game, played between Sarah and Paul.
Sarah's moves are indicated by solid arrows (——►)
Paul's moves are indicated by dotted arrows (- - -►)
It is Sarah's turn. She has two possible moves.
Show that from one of these moves Sarah can ensure that she wins, but from the other Paul can ensure that he wins.

(ii) If the game is played from the beginning and Sarah has the first move, then she can always win the game if she plays correctly.
Explain how Sarah should play in order to be sure of winning.

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which contain a list of strategic hints. It is intended that these "checklists" should only be offered to pupils who are in considerable difficulty or later as a stimulus for reflective discussion. The problems in this Unit respond to similar strategies to those introduced in Unit A, but begin to vary in style. In particular, one task involved the strategic analysis of a simple game. (See opposite page) Unit C is built around three tasks which differ in style, but which again respond to similar problem-solving strategies. No printed guidance is offered to pupils, but the teacher has a "checklist" of strategic hints which may be offered orally to pupils in difficulty. Finally, the support materials aims to provide a 'do-it-yourself' inservice training resource. These materials are divided into five chapter headings: "Looking at Lessons;" "Experiencing Problem Solving;" "How Much Support do Children Need?;" "How Can the Micro Help?;" and "Assessing Problem Solving." Each of these chapters suggest activities, some only involve the teacher in looking at the material, some suggest trying something with another class, while others require a few teachers to get together to watch videotaped lessons and discuss their implications.

We hope that the gradual introduction of teaching modules, such as the one described above, will provide the motivation, resources, and support necessary to enable teachers to develop their teaching styles without feeling that 'shock' which often accompanies more sudden, radical innovation. Our initial trials lead us to be optimistic.

THE "FIRST TO 100" GAME

This is a game for two players.
Players take turns to choose any whole number from 1 to 10.
They keep a running total of all the chosen numbers.
The first player to make this total reach exactly 100 wins.

Sample Game:

Player 1's choice	Player 2's choice	Running Total
10		10
	5	15
8		23
	8	31
2		33
	9	42
9		51
	9	60
8		68
	9	77
9		86
	10	96
4		100

So Player 1 wins!

Play the game a few times with your neighbour.
Can you find a winning strategy?

- Try to modify the game in some way, e.g.:
  - — suppose the first to 100 *loses* and overshooting is not allowed.
  - — suppose you can only choose a number between 5 and 10.

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THE "FIRST TO 100" GAME . . . PUPIL'S CHECKLIST

Try some simple cases	• Simplify the game in some way: e.g.:— play "First to 20" e.g.:— choose numbers from 1 to 5 e.g.:— just play the end of a game.
Be systematic	• Don't just play randomly! • Are there good or bad choices? Why?
Spot patterns	• Are there *any* positions from which you can always win? • Are there other positions from which you can always reach these winning positions?
Find a rule	• Write down a description of "how to always win this game". Explain why you are sure it works. • Extend your rule so that it applies to the "First to 100" version.
Check your rule	• Try to beat somebody who is playing according to your rule. • Can you convince *them* that it always works?
Change the game in some way	• Can you adapt your rule for playing a new game where: — the first to 100 *loses*, (overshooting is not allowed) — you can only choose numbers between 5 and 10. — . . .

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WHO INTERPRETS AND TRANSFORMS RESEARCH?

Arthur Clegg, Assessment of Performance Unit
England

The Assessment of Performance Unit (APU) was set up in 1975 within the Department of Education and Science to promote the development of methods of assessing and monitoring the achievement of children at school, and to seek to identify the incidence of under-achievement. It has conducted annual surveys in Mathematics at age 11 and 15, 1978-82; in English Language at age 11 and 15, 1979-83; in Science at age 11, 13, and 15, in 1980-84. Foreign Language is being surveyed at age 13, 1983-85.

WRITTEN REPORTS

Until 1983, the research was reported year by year at each age level in a large written report of about 150 pages, e.g.:

Mathematical Development Primary Survey Report 1, 2, and 3
Mathematical Development Secondary Survey Report, 1, 2, and 3
Language Performance in Schools Primary Survey Report 1 and 2
Language Performance in Schools Secondary Report 1 and 2
Science in Schools Age 11 Report 1
Science in Schools Age 13 Report 1
Science in Schools Age 15 Report 1

These documents were circulated to academic libraries and to each of the regions (local education authorities) but schools had to buy any further copies at about 6 or 8 each. Few teachers read the documents: they were expensive and written for a wider audience than teachers alone and few teachers came across them.

A new publications policy was launched in lage 1983 when it was decided to have three kinds of publications in addition to a newsletter.

Full Research Reports. These reports were to be produced and issued free on the same restricted circulation as for earlier reports. We have produced:

- Science in Schools Age 11 Report 2 (1983)
- Science in Schools Age 13 Report 2 (1984)
- Science in Schools Age 15 Report 2 (1984)
- Language Performance 1982 Primary Survey (1984)
- Language Performance 1982 Secondary Survey (1984)
- A Retrospective Report on Mathematical Development 1979-82 Surveys (1984)

Reports for Teachers. These reports are pocket sized about 20 x 14 cm and about 40 pages long. All schools are issued with one free copy and larger secondary schools get two. Additional copies may be bought. We plan to produce about ten before 1985 (April). The first few are published - Science at Age 11, Science at Age 13, Framework for Science at Age 11, Framework for Science at Age 13-15. Reports on English Language are coming: a framework for the assessment of language, assessing writing and assessing oracy. The Mathematics reports for teachers will cover a range of topics and start to appear in the autumn 1984. (The energies of the mathematics team are at present devoted to the full research report 1979-82).

Occational Papers. These papers are written by named individuals on an aspect of the research findings:

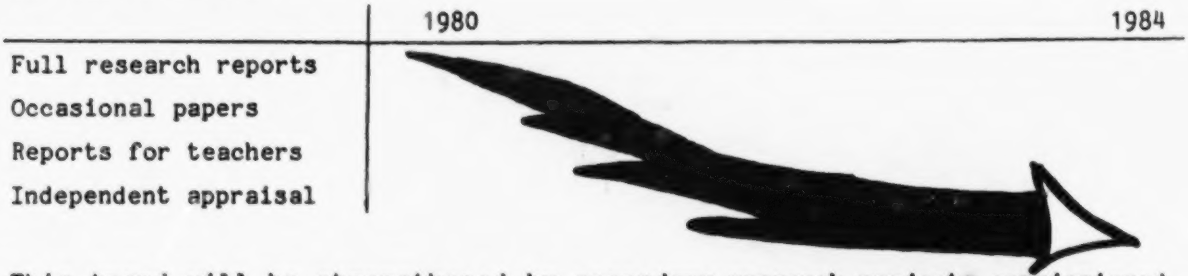
- Learning Mathematics - How the Work of the APU Can Help Teachers - J.S. Eggleston
- Foreign Language Provision - by Monitoring Services Unit, National Foundation for Educational Research
- Expectations and Reality - A Study of the Problem of Interpreting the APU Science Surveys - Black, Harlen, Orgee
- Performance of Boys and Girls - in draft

In addition, the members of the teams write for academic journals and in one of the English mathematics teachers' journals (Mathematics in Schools).

The early full research reports state the facts; they describe the tests used, the assessment framework, the statistical design of the test and details of pupils' responses. There is also some analysis relating mean scores for domains to other variables such as school location, curriculum, class size, etc. Essentially the reports describe and measure what is and refrain from expressing opinion about what pupils should be able to do. The later reports make what might be called 'reasonable speculations', drawing attention to particular features of performance, revealing common errors or weaknesses such as using indices as multiplying factors, confusing perimeter and area, applying false strategies when putting decimal fractions in order of magnitude, generally finding estimation difficult and often not being able to develop good strategies for measuring dependent variables in investigations.

The reports for teachers have selected the findings which are of prime interest to teachers and have speculated a little but have stopped short of commenting about what should be the level of performance. The occasional papers written as they are by named individuals have perhaps exercised a little more academic and professional freedom in commenting on the data they have selected for presentation.

We see in the period from 1980 (publication of the full report of the first Mathematics survey) to 1984 with the publication of reports for teachers and occasional papers a movement towards picking out material of significance for the classroom.



This trend will be strengthened by secondary research projects commissioned by the Department of Education and Science.

Independent Appraisal of the Significance of APU research for teachers in the classroom - MATHEMATICS. This is being conducted by the Cambridge Institute of Education and Cambridge University and a report is expected in 1985.

Independent Appraisal of the Significance of APU research for teachers in the classroom - LANGUAGE (English). A report is expected in 1985.

Children Learning in Science - is following preliminary findings in science from APU with indepth studies about children's understanding of science concepts. About 9 conceptual areas in science will be explored. The reports and the short reports for teachers makes explicit reference to what teachers might do about the findings. The first of these reports, already published, is Aspects of Secondary Students' Understanding of the Particular Nature of Matter (Leeds University, Brook, Briggs, and Driver, 1984). Phase 2 of this research is part of the big curriculum Review in Science (Director, R. W. West) which is funded by the School Curriculum Development Committee. Teachers groups will work with the Children Learning in Science team to develop classroom strategies which take account of research findings.

It is envisaged that an independent appraisal of science will be commissioned in 1985.

The foregoing has indicated how the written produce of the APU research and the secondary research stemming from it has shown a positive trend towards what might be called in other contexts 'user-friendliness'.

CONFERENCES AND WORKSHOPS

In 1982-83, as the data from the final year Mathematics survey was being analyzed, the APU held a series of six regional conferences. The purpose was to explain to senior administrators, advisors, and heads what stage APU had reached and to indicate that the data collected, and the instruments designed to fit the clearly described assessment frameworks, had potential benefit for practicing teachers and for school administration. The message to the Unit from conference participants was clear: first, short reports specially written for teachers were needed; but second, personal presentations were essential, backed wherever possible with good videotapes.

As a result of the conferences, the two inspectors attached to the APU central unit and the research teams were asked for many talks and as time went on, the requests for talks became requests for workshops. The modes of testing which we call 'practical', i.e. which are really "clinical" interviews between a trained tester and a single student, have created a great deal of interest.

During the period September 1983 to July 1984, the two inspectors from the central Assessment of Performance Unit will have spoken in 50 different locations covering about half the local education authorities in England and Wales to a total audience of about 4,000 headteachers and/or advisors. The research teams have collectively matched this effort - whilst an inspector was in one place there was usually a member of the research team helping with a workshop in another.

WHO INTERPRETS AND TRANSFORMS RESEARCH?

The culmination of the new publication policy and the conference programme 1983-84 has been to raise the profile of APU. The leaders of curriculum groups and advisors and many heads are aware that we exist and have something to offer. The presentations given by APU have been popular and entertaining, so now we have created a demand which we cannot satisfy. This is a total reversal of attitude in a decade. When the first idea of a central government agent monitoring standards was mooted it was regarded with some hostility and suspicion. Now teachers are ready to listen.

Who should tell them?

What should be told: research findings, some reasonable speculation about their meaning or should the curriculum significance be strongly stated perhaps with some suggestions for teaching?

A round of specialist conferences is designed to harness the energy of teachers who have been involved in the research. Inservice workshops in universities and local schools are beginning to build in activities based on APU research. Examiners are talking to us. We are working on mechanisms to give others access to our databanks.

The research teams are no longer tied to a pattern of annual monitoring: surveys will now take place at 5-yearly intervals. Between surveys, the teams, in addition to pursuing further resarch, will design products to feed into inservice networks to assist teachers to develop insight into pupils' performance and its assessment. The research teams will strongly influence the message but other agents will have to do most of the telling.

EFFECTS OF RESEARCH ON TEACHERS

USING THE MATHEMATICS OF LOGO AS A SPRINGBOARD FOR THE TEACHING OF ELEMENTARY SCHOOL MATHEMATICS

Carolyn Kieren, University of Quebec a Montreal
Canada

This paper looks at the reports of several Logo research projects which have described not only Logo environments, but also the kinds of mathematics which children (aged from 8 to 13 years) use in these environments. The implicit assumption of this paper is that an elementary school teacher with a Logo-equipped microcomputer in her classroom could build upon the mathematics experienced by her pupils within the Logo environment and use it as a basis for introducing some of the more traditional elements of the school maths curriculum, thereby further enriching the already-rich Logo experience. As a first step in that direction, this paper presents some researched ideas on the setting-up of a Logo environment and recounts the mathematics experienced by children in this kind of environment.

A LOGO ENVIRONMENT

There is a dilemma inherent in using Logo in the classroom. It is that Logo, and here we are referring to the turtle graphics part of the language, was intended by its designer (Papert, 1980a) to be a microworld which a child should explore with directed instruction. This puts the classroom teacher in an unusual situation — how to utilize Logo in teh classroom without actually "teaching" it. A second problem is how to link up the experience gained in Logo programming with the school math curriculum. A third problem exists, but it is an economic one. It would be ideal if each child could have his own computer, or if the classroom was equipped with at least one computer for each two children. However, since we are far from the ideal, most teachers would settle for at least one computer in each of their classrooms.

But then what can an ordinary elementary school teacher hope to accomplish with one computer installed in her classroom? Most of the Logo research projects which have been carried out in schools up to now have used the resources of someone other than the regular classroom teacher. The guidance offered to each child has usually come from the researcher, or from the computer resource person of the school with the researcher(s) observing. Furthermore, must of the research has taken place in a computer lab of the school, not in the classroom. However, there are a few notable exceptions.

The Chiltern Logo Project. The Childtern Logo Project (Noss, 1984) set up a team of five elementary classroom teachers, none of whom had had any prior expertise with computers. The first six weeks were spent famailiarizing the team with Logo and the ideas surrounding it. The main priority of the project was to uncover the ways in which children's mathematical and heuristic ideas develop as they learn to program. Each classroom was set up with a computer, a floor-turtle, a printer, and a version of Logo. The children of the study (who seemed to range in age from about 8 to 11 years) were of mixed ability. They worked in groups of two or three (although two was found to be a preferable number from a learning point of view) for one or two sessions per week (about 75 minutes in total per week) throughout the year. It seems important that each child get to the computer at least once a week (Bert, 1983), but if possible, more often than this in order for significant learning to take palce. In each classroom, a group was engaged in Logo activities "at the back", while the rest of the class continued with their normal work. Despite limited resources, they attempted to build a Logo culture within the classes -- an atmosphere in which programming ideas were discussed and in which other curriculum work was often linked to Logo work. The teaching strategy adopted was an unstructured approach in which the teacher's intervention was restricted largely to informal advice and suggestions. The policy of minimal intervention allowed them to gain some insights as to instances when intervention works:

> 1. the child has already tried out her solution to a problem unsuccessfully;
> 2. she expresses (explicitly or imlicitly) a need for more power;
> 3. the child needs "just a nudge" to get started, perhaps a reminder of an idea or a suggestion for an approach;
> 4. a new idea would be welcomed by the child because it would connect with other ideas the child is familiar with. (Noss, 1984, p. 150)

They also gained insights as to how the teacher should intervene -- as gently and unobtrusively as possible. The following worked:

> 1. Offering a short prewritten procedure which illustrates a new idea (say, recursion). It is important that the child can, if she so wishes, "look inside" the procedure to understand and modify it.
> 2. Review a piece of work with a child in order to encourage her to modify it, generalize from it, or otherwise improve it.

> 3. Suggesting a "challenge" which may illustrate a particular idea, or may lead the child to perceive the need for a particular idea.
> 4. Helping a child to plan a project -- often one which relates to other classroom activity. (Noss, 1984, p. 150)

A further suggestion comes from Berdonneau and Dumas (1981): though a teacher might feel more comfortable with having the entire class work on the same project at the same time, a diversity of projects progressing simultaneously obliges the teacher to adopt a position which is much less directive, but giving her also the opportunity to be, side by side with her pupils, in a true learning situation.

The whole question of intervention is a delicate one. Many Logo advocates claim that teacher intervention of any kind fiolates the spirit of discovery learning which is supposed to be part and parcel of the Logo environment. ut this is not actually so. In the Brookline research project which involved Papert and his MIT colleagues (Papert et al., 1978; Papert et al., 1979; Watt, 1979), the children were virtually bombarded with various programming suggestions from the observers and participants in the project. It was also clear that each of these same children chose to adopt or not adopt certain ideas proposed to them. They used whatever suggestions they felt ready for or wanted, at the time.

Direct Mode vs. Programming Mode. This personal selection by the children of various suggestions offered by others applies also to their mode of computer utilization. The Chiltern study emphasized the programming mode (i.e., teaching the turtle new words, and then using these new words as subprocedures). However, many children, especially the younger ones, seemed to prefer working in the direct mode (i.e., immediate execution of each Logo line; no use by the child of subprocedures). Noss states that the children found it easier to debug in the direct mode.

Emphasis on the direct mode was an essential component of a study carried out in France (Bideault-Delavenne, 1983) with 24 children (aged 8-10 years) over the course of 12 sessions. The children were never introduced to the programming mode; they worked exclusively in the direct mode. This was a conscious decision on the part of the investigator, for prior pilot studies had indicated that the children needed the immediate feedback provided by the direct mode. Bideault-Delavenne claimed that the programming mode required elaborate mental representations involving perception, memory, mental images, and language which were beyond the capacities of the children of her study. Her results, of which more will be said later, showed that extended experience with the direct mode (which provided immediate feedback from the computer) allowed these children to get a good hold of the quantitative aspects of number which had served as inputs for RIGHT, LEFT, FORWARD, BACK. Furthermore, according to the author, the immediate feedback of the direct mode helped the children adopt an attitude of "hypothesis-verification".

The controversy over whether children should spend a lot of time in the direct mode before being exposed to the programming mode is probably best put into perspective by looking at the results of a study by Rampy (1984). She identified the programming styles of 12 fifth grade students learning Logo over a 6-week (105 minutes per weekly class) Saturday morning course. She found that, "given a choice, students will select programming tasks that differ in structure, complexity and amount of detail. Some students will require long periods of uninterrupted work at the computer to complete a desired product; others will choose to explore a process for only a short time before seeking to alter that process. The projects on which students in this study worked were self-defined and could have been abandoned at any time. Yet the product-oriented students were persistent in solving problems tha arose." (p. 10)

According to Rampy, the product-oriented students began by defining their task, sketched it on paper, worked in the direct mode, corrected their bugs, and never gave up until their picture was "right". These students used visual clues rather than knowledge of mathematics to complete shapes, and generally solved their problems through trial and error. The short instructional sessions which were a part of this study seemed to have little immediate effect on the work of these students. Although they eventually tried out what had been introduced in the group instructional session, they never abandoned a plan on which they were working to attempt something new. Rampy states further that "these students did not appear tob e slow to understand new commands and procedures, only slow to attempt something new....their primary objective was to complete their self-defined project." (Rampy, 1984, p. 8)

The process-oriented students, on the other hand, preferred to experiment with a variety of commands and procedures. They would generally begin work by defining a procedure and experimenting with various inputs; they did not appear to have in mind a particular design they wished to achieve, rather to explore the power of the procedure they had defined. Occasionally, according to Rampy, the process-oriented students would develop a plan based on what their procedure seemed to suggest, but they appeared quite willing to alter that plan if a bug offered a new idea. Because these students did not invest a great deal of time in any one procedure, they often abandoned a project if it did not quickly produce interesting results. "While the product-oriented students made few pictures but saved all of them, the process-oriented students made numerous pictures and designs but saved relatively few of them." (Rampy, 1984, p. 9)

What all of this suggests is that it seems best to have as rich a Logo environment as possible, but that not all children will use what is available in the same way. Some prefer to work in the direct mode, some in the programming mode. But what is important, according to all of these studies, is that each child be free to pursue his or her own learning within the Logo environment. The teacher has a vital, yet subtle, role to play in helping children to learn in this environment. We now look at the kinds of mathematics which dhileren experience within a Logo environment and suggest that the teacher might further enrich this learning.

MATHEMATICS EXPERIENCED IN A LOGO ENVIRONMENT

Mathematical Thinking. Noss (1984) characterized the kinds of mathematical thinking which were fostered by the Logo learning experience among the children of the Chiltern project:

> 1. We were impressed by the way in which the process of learning Logo encouraged the twin activities of generalization and particularization. Ideas like "It'll work for other shapes now" on the one hand, and "Let's try an example" on the other, became familiar to most children. In addition, children became adept at switching from one kind of thinking to the other.
> 2. The activity of Logo programming encouraged an atmosphere of conjecture within the most programming groups. This took the form of a) What if?, and b) How? The former was associated more with exploration (I wonder what would happen if...?). The latter was more a characteristic of solving problems (How can I get the turtle to draw this?)
> 3. Our findings suggest that Logo does encourage children to look for and believe in the existence of underlying rules and theorems. It seemed evident that such an awareness was generally not present at the beginning of the work, and was gradually built up during the year. This is not to say that the children "learned theorems", still less that such knowledge transfers to the rules of school

> mathematics. On the contrary, it may be more powerful than this. Understanding the idea of a theorem is a prerequisite for understanding any particular theorem. (p. 148)

Mathematical Concepts and Properties. The report of the Brookline Logo Project, referred rto earlier, provides us with a detailed description of the matehamtical concepts associated with the programming activities of 16 mixed-ability sixth grade students. The first hine items outlined below are derived from the Brookline report; the remainder, from other reports.

1. Qualitative Structuring of the Number Worlds. The use of numbers as inputs to turtle commands required the children to recognize the different roles for numbers within turtle geometry, for example, FORWARD 50 vs. RIGHT 50. In producing figures, the input to FORWARD determines the size of the figure, while the input to RIGHT determines the shape. As an imput to FORWARD, a bigger number produced a "bigger" effect; while, as an input to RIGHT, a bigger number usually produced a "different", but not necessarily "bigger", effect. Splitting the "world of numbers" into "length numbers" and "angle numbers" provided a qualitative structure for that world.

2. Quantitative Structuring of the Number Worlds. Estimating the practical effects of particular numbers provided a quantitative structure for the world of number. A child's first quantitative structuring of numbers in the Logo world often occurred, according to Papert et al. (1979), when she became aware of certain limiting factors and realized that certain numbers were too small or too large to be of practical effect in most applications. The children of the Brookline study also developed strategies for estimating the number of turtle steps needed to mvoe the turtle to a particular point on the screen. The estimate was often refined by an approach involving successive approximations. They also developed strategies for estimating the amount of rotation necessary to aim the turtle in a particular direction.

3. Certain Properties of the Number Worlds. The children used " composition" (e.g., the additive property of numbers) when the combined turtle commands, such as, FORWARD 25, FORWARD 25, combined as FORWARD 50. They used "inversion" (e.g., formation of the inverse or negative of an operation) when they were able to use BACK as an inverse to FORWARD and LEFT as an inverse to RIGHT. The combination of these properties was seen when children aggregated a series of commands, such as, FORWARD 30 BACK 10 FORWARD 5 into one command FORWARD

25, or a similar series with respect to rotation, LEFT 90 RIGHT 10 LEFT 5 into LEFT 85.

4. The Use of Coordinate Systems. Through their work on specific projects, the children came to use global coordinate systems of their own. They used these systems to solve problems that required that they take into account aspects of geometry other than the turtle's immediate position and heading. Some of the systems they used, without necessarily being aware that they were using them, according to the Brookline authors, were domain specific or intrinsic coordinates, various types of polar or angular coordinates, and standard cartesian coordinates.

5. The Use of Heuristics. While solving their own problems, the children began to discover some of the regularities of the matheamtical world in which they were functioning. Such regularities were used by the children as heuristics — strategies or rules of thumb that are helpful in problem solving. Heuristics used by the children of the Brookline project included breaking a large problem into smaller more easily solved parts, "playing turtle" — to figure out which way to move the turtle in a specific instance, and repeating a shape until an interesting design occurred.

6. The Significance of 360 Degrees. Some children quickly realized that when repeating a shape and a particular rotation, certain angles produced fairly simlpe closed figures, while other angles "filled up the screen" before closing. When they began to focus on the particular angles which made the simpler shapes, they began to realize the significance of 360 degrees.

7. Construction of Equilateral Triangles and Other Regular Polygons. Once they had drawn a square with the turtle, many children went on to try a triangle. Though it is fairly easy to do this by trial and error, the approach derived from the process of constructing a square is far from trivial. In the Brookline study, the teacher worked together with the children on this process. Some of the children then explored the generality of this approach by trying to construct 6, 8, or 10 sided regular polygons.

8. The Use of Similarity. In a Logo environment, children encounter and make use of similarity in a number of ways. A proportional change in all the FORWARD and BACK steps in a sequence of turtle commands, while holding the angles constant, changes the size, but maintains the shape of the figure drawn by those commands. While few students of the Brookline study came to

understand this principle in its full generality, there were many ways in which students encountered it in simpler forms and used it in their Logo projects. The desire to create similar designs often provided children with their first use of variables as they tried to create "different sized squares" (using inputs to procedures). This seemed quite a natural introduction to what is often considered a difficult concept to use in high school algebra.

9. The Use of Symmetry. In the Brookline project, most children encountered the idea of symmetry as part of their Logo experience. According to Papert et al. (1979), a Logo symmetry theorem might be: "If all the right and left commands in a sequence of TURTLE commands are reversed, without changing any of the other commands in the sequence, the resulting design will be a mirror image of the original design." (p. 5.70) The reversing of RIGHT and LEFT is one approach the children used to create symmetrical designs. Another was the use of an implied axis of symmetry, usually a vertical line down the center of a design, in which both sides were identical but in which the symmetry was produced by working across from one side of the design rather than by starting from the middle and reversing RIGHT and LEFT commands.

10. Use of Rotation and Translation. A study carried out by Shultz (Shultz et al., 1984) involving 37 children in grades 5, 6, and 7 (aged 11-13 years) in a Logo experimental class for 30 45-minute sessions throughout the year aimed at assessing the impact of learning Logo on the acquisition of a variety of logical and mathematical concepts. It was found that the children acquired a notion of rotation — a closed figure is turned about a point without altering the size or shape of the figure and without otherwise moving the figure; and also a notion of translation — a closed figure is slid to another position without rotation or changes in size or shape.

11. Operations on Numbers. A year-long study carried out in France (Berdonneau & Dumas, 1981) with a class of fifth graders (about 11 years of age) reported the finding that the children were experimenting with an operation as input to FORWARD and BACK, for example, FORWARD 13+20, followed by BACK 33, which returned the turtle to its departure point. These children also tried out commands such as RIGHT -30 and RIGHT +30. They also calculated mentally quite frequently, especially when using the properties of composition and inversion, an indication that, according to Berdonneau and Dumas, the use of computers does not cause children's counting and number skills to atrophy.

Ability to Talk About Their Mathematics. Two studies in particular have pointed out that experience with Logo is especially helpful in developing children's ability to talk about their mathematics. In one study, Bideault-Delavenne (1983) worked for 4 months with a class of third graders (aged 8-10 years). Her aim was to uncover children's strrategies in a Logo environment and the transfer of these strategies to other non-Logo situations. During the Logo sessions, the children were encouraged to talk about what the turtle had done and also to predict what the turtle would do. One of the results of the study was that the children of the Logo group were able to express their ideas of measure and distances in a non-Logo task much more clearly than the children of the control group. The Logo children used a more precise vocabulary and were able to easily explain the "how" of their actions, something which the control group had a great deal of difficulty in doing.

Another study with results along the same lines is that of Howe and his colleagues in Edinburgh (Burns, 1982; Howe, 1982; Howe, O'Shea, & Plane, 1980). They used Logo in their first laboratory study as a vehicle to improve the mathematics achievement of average and below-average 11-12 year old boys. What is interesting about this study is the finding that, as a result of their Logo experience, the boys "could argue sensibly about mathematical issues, and could explain mathematical difficulties clearly" (Howe, 1983, p. 16), something which the control group boys were unable to do.

CONCLUDING REMARKS

The above findings indicate that many mathematical ingredients exist in a Logo environment. But can more be done with them? We suggest that the mathematics concepts with which a child becomes acquasinted in a Logo environment can all be brought out further and developed more deeply. All of these Logo experiences can be further enriched by bringing in topics from both the existing school mathematics curriculum and also from teh outside world. For example, the Logo experience with angles and triangles can be supplemented with discussion on various classifications of angles and triangles. The Logo experience with symmetry can be linked up to observations of symmetry in nature. What we are suggesting is that Logo can serve as an initial point of discovery, but that afterward these discoveries could be discussed and supplemented by other materials -- yet, always trying to relate these other materials to what was done in the Logo environment.

The teacher's role in this integrative process is not a simple one. However, by beginning with the Logo experience and then enriching it with outside materials rather than following the reverse order, that of beginning with the traditional curriculum and then trying to fit Logo to it as a means of enriching the current curriculum, we allow what might be the mot powerful tool to be at the basis. In this way, the *doing* of mathematics in a computer environment becomes a vehicle for the learning of mathematics. If children learn by doing, then an enriched Logo environment is a way "in which one might be able to put children in a better position to *do* mathematics rather than merely to learn *about it*" (Papert, 1980b, p. 177).

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THE RELATION BETWEEN RESEARCH AND BOTH PRESERVICE AND INSERVICE ELEMENTARY SCHOOL TRAINING

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In France, as in the majority of developed countries, educational research was for a long time insulated from everyday practice within the educational system. However, from the middle of the 1970s onwards, conjointly with the general acknowledgement that the school system was going through a crisis, that it was the source of too much educational failure and that it no longer reflected the expectations of the younger generations, a will for change emerged. This will for change was accompanied by a new aspiration to integrate research within this general innovatory movement and, more particularly, in the field of teacher training. Over the same period, there was a development in research in didactics and more especially, in mathematical didactics under the impetus from the Research Institutes for the Teaching of Mathematics (IREM), and at a somewhat later date from research teams in certain universities, particular: Bordeaux, Grenoble, Marseille, Nancy, Orleans, Paris, Strasbourg... (Chevallard, 1981).

At presenter there is in Grenoble: A Research Unit responsible for the didactics of mathematics, and an IREM, whose main task is inservice teacher training and which is part of the Teacher Training Institute. This Institute, whose principal functions are preservice and inservice training of secondary school teachers and educational research is also responsible for the university training of elementary school teachers. The aim of this paper is to show the changes that have taken place over the last ten years in the relation between research and training in this sector.

A BRIEF OVERVIEW OF THE RELATION BETWEEN RESEARCH AND TRAINING IN THE GRENOBLE DISTRICT

The Years 1970-1975. In the 1970s during the big reforms in the teaching of mathematics in the elementary school, the first research into basic learning at the elementary school was sparked off by various different factors: problems arising from everyday classroom experience, obstacles encountered either by teachers or learners during the application of certain innovations.

These beginnings were facilitated by the fact that the IREM of Grenoble was already interested in elementary schooling. This meant that it was possible to obtain backing for the creation of mixed research teams of university researchers, lecturers in training colleges, teachers in secondary schools and in elementary schools.

The research teams' experimental work was made all the easier by the fact that the aims corresponded exactly to the needs expressed by the teachers themselves and thus it was favourable received both by the administration and the schools. These research teams were made responsible for the introduction of new contents and methodology, for their analysis and for the organization, as well as for the management of inservice training within the same field.

It was these first few years of active involvement with the practical problems that fostered the emergence of productive research subjects, oriented towards the learning of fundamentals in the elementary school, while at the same time research into mathematical didactics at the national level was beginning to forge its own identity.

The years 1975-1980. Mathematical didactics research was characterized in Grenoble during this period by: Research projects, relating to acquisition of the notion of natural number (Bessot, Comiti, 1978; Comiti, 1980; Comiti & Company, 1980); researchers assuming responsibility for relevant learning tasks in collaboration with teachers; the extension of experimental work to other areas within the classroom, for example, to decimals (Comiti, Neyret, 1979), geometry (Guillerault, Laborde, 1980), measurements (Eberhard & Company, 1979) and the use of calculators (Croquette, Guinet, 1979); the continued organization of advanced training courses (teacher inservice training); the publishing for a regular progress report for primary school teachers, the journal "Grand N", in order to encourage further debate and discussion; and the organization of inservice training courses for the benefit of teacher trainers. All these factors, taken together, created the right conditions for the emergence of an authentic momentum involving, at the same time researchers, trainers, and the teachers themselves.

The years 1980-1984. It is over these last few years that we have simultaneously witnessed: A deepening at the level of the nature of the questions that are being asked in research and a diversification of the research areas (number, geometry, measurement...); a re-examination of certain innovations and experiments carried out in the 70s. This has been linked, on the one hand, to conclusions stemming from research results, and on the other hand, to the analysis of learning behavior within the classroom situation during the previous years; an increase and diversification of inservice training and of circulation of progress reports (Balacheff, Neyret, 1982; Bessot & Company, 1982; Bessot, Eberhard, 1982; Bessot, Comiti, 1981; Bessot, Comiti, 1982); and the elaboration of new contents and methods for initial elementary school teacher training, involving researchers in specific aspects of initial training (recent reforms having made teacher training the responsibility of the university).

It should be noticed that it was the progress in research, both at Grenoble and at the national level, that was at the origin of this development in initial training: it was due to the fact that it fostered the elaboration of a body of knowledge relative to the teaching and the learning of mathematics within the classroom situation (Brousseau, 1981).

All this, of course, is closely linked to the general development of research into didactics (Artigue, Robenit, 1982; Audigier, Cauzinille, 1979; Audigier & Company, 1982; Balacheff, 1983; Brousseau, 1980; Columb & Company, 1980; Vergnaud, 1979 et al., 1982a) and to the elaboration of theoretical frameworks for this research. At the national level, this has been facilitated by: The organization of a national seminar in mathematical didactics where different approaches can be compared and results validated; the publication of a journal: "Recherches en Didactique des Mathematiques"; and the creation of a national research unit in association with the CNRS (The National Center for Scientific Research). This unit is codirected by G. Vergnaud and G. Brousseau.

MATHEMATICAL DIDACTICS IN FRANCE TODAY

The development of mathematical didactics in France has taken place especially over the last ten years and this has been accompanied by the elaboration and the refining of the theoretical framework, notably the theory of didactic situations (Brousseau, 1978, 1984). Briefly, this theory states that given knowledge is part of a "conceptual field" which may be structured in situation classes (Vergnaud, 1982b). A model can, therefore, be elaborated of different behaviour pattern patentials (a priori analysis) and the significance observed behaviour can be studied in relation to the situation classes and identified by learning that has taken place, whether it be stable or transient. Didactics is based on a theory of knowledge and concept formation which holds that teaching and the communication of knowledge are themselves part of concept formation.

It follows from this that the role of didactics is to apprehend knowledge via the conditions in which it becomes manifest, so that these conditions can be reproduced (at least approximately) in order to activate a meaningful and a functional learner acquisition process.

The specific interest of this research and what explains that it both fits into the existing paradigm and differentiates itself from other approaches to the problem, resides in the fact that the three main situational constituants are taken into account, namely: *what is being taught*, *the learner*, and *the teacher*. The sorts of questions that didactics is trying to answer are concerned with these three components and their interaction within the framework of a teaching system. Fundamentally, the questions are the following: What are the different conceptions that the learner develops about a given notion?; What are the tasks that the learner should be confronted with, so that his knowledge system might develop?; What are the conditions (didactic, psychological...) that must be united so that knowledge can be transmitted to and acquired by the learner?

All this requires the analysis of classroom situations and of learner behaviour but equally the analysis of teacher decisions and of the interctive process between teacher and learner relative to the objects of knowledge (Chevallard & Company, 1983). These analyses enable the different conceptions underlying the learners' reasoning, at a given moment, and within a given situation, to be demonstrated; the construction of learning situations which will foster an evolution and an expansion of the learner's conceptions; the location of the significant variables within a teaching situation in order that they might be reproduced on a scientific basis.

THE ROLE OF THE GRENOBLE RESEARCH TEAMS IN THE IMPLEMENTATION OF A REGIONAL ELEMENTARY TEACHER TRAINING PROGRAM IN MATHEMATICS

The Grenoble team have developed their own experimental methods on which they have based their research. Essentially these consist of: First, interactive situations wehre the task of the learners is to solve a problem as a group (Balacheff, 1983; Guillerault, Laborde, 1984). Learner interction which is the central factor of this approach, allows to obtain a record of the origin of the written formulation that is finally adopted. This learning interaction produces a decodable linguistic formulation of the analysis of the problem and of the choices made in its description. The resulting conflicts in opinion that may appear lead the learner into operations of validation in order to reach a group consensus. Second, didactic situations in which the researcher takes an active part together with the teacher, in a teaching process in a school context (Bessot, Eberhard, 1984; Comiti, 1983). In such cases, it is the didactic variables (that is to say the situation types) which are of fundamental importance, as it is they which command, and thus allow, the experimenter to vary the differences in the formulation of knowledge types. These interactions, whether they be interactions between learners or between the learners and the teacher, are in themselves, part of the variables.

It is thus clear that the aim of this research is not the production of model lessons but the building up of a body of knowledge relating to teaching and learning of mathematics within the framework of the school. The results that have been obtained mean that we have a considerable role to play in catering for the specific needs in teacher training. In what follows I will restrict myself to our role in preservice elementary school teacher training, but we also play an important role in the training of secondary school teachers (Balacheff, 1984).

As a result of the confrontation and comparison of our research findings and our experience as inservice trainers, we have come to believe that, unlike what happens for the most part in France, teacher training in mathematics cannot be separated into two parallel, or even worse, consecutive, training periods: one being purely mathematical and the other being purely pedagogical. This is because the study of teaching is an activity and a field of knowledge in itself, from which no component can be excluded. This having been established we define the principal specific objectives of primary preservice teacher training as follows: it should enable teachers to choose or to elaborate activities adapted to their students and which will facilitate conceptual acquisition; to have a varied approach in the management of mathematical activities; to evaluate the results obtained by the learners; and thereby, be able to audoevaluate their own teaching performance; to analyze and understand phenomena within the teaching process and to locate problem areas: and to adapt their choices and their techniques relevantly in order to improve their results.

The training program that we are trying to set up in Grenoble for the teaching of the Primary School Mathematics major which caters for 200-300 student-teachers, depending on the year, half of whom major in mathematics, is based on the acquisition and the practical manipulation by the student-teachers of mathematical concepts in situations which are either purely mathematical or maths dependent. It is also based on the study of the conditions in which the learners acquire concepts; that is to say, the study of the conditions in which the learners acquire these concepts. It follows from this that we try to supply the student-teachers with problem situations to be explored, so that certain fundamental concepts ay be better acquired (in particular, numeration, arithmetic, the extension of natural number, numerical functions, geometry, etc...); to study, analyze, and capitalize on the student-teacher's own attitude, faced with a mathematical situation; to specify the components of a theoretical didactics which would permit the analysis of the conditions in which mathematical knowledge is revealed and acquired; to place the student-teacher in the practical conditions of such a study by setting up different teaching situations and then by working them through, observing and analyzing them; and to carry out a critical examination of the criteria governing syllabus content in the elementary school. (Syllabus analysis, its evolution, the history of concepts and the way in which they have been taught...)

An example of progression in such a training program. This progression takes place during the second year of preservice-elementary school teachers. Twenty-four student-teachers work together; they have two mathematics trainers who work with two elementary school teachers within the classoom of whom student-teachers can go and conduct sessions (1). Here is a summary of this progression.

I - Theoretical work about natural numbers, addition, subtraction, multiplication, multiples.

II - Preparing a sequence about "magnets"

1. A priori analysis: each student-teacher is given two word problems, A and B (2). They have to compare each of these situations to the other and to choose one of them: they choose B.

2. Analysis of pupils' productions: the student-teachers have to analyze children's protocols realized last year with the situation B, in teh same level class, at the same moment of the year.

3. Preparing the class session: the student-teachers have to decide what kind of instructions they will give to the children; the way to start in the situation; what different types of intervention will be able to help the pupils;

what procedures are expected; what links will be between procedures and results.

III - First session within the classroom

The 24 student-teachers are divided into four groups of six. Each group take the responsibility of half-class (ten pupils). In each group, one student-teacher conducts the session, the five others make observations (two pupils per observer). AFter the session, the student who has conducted the session has to do a report about how the session progressed; the others have to prepare an analysis of the pupils' procedures with their observations.

IV - Preparing a second session

First, there is a collective discussion: The student-teachers study the pupils' procedures they have observed. They examine the different possible tracks to continue; several of them are proposed; the final decision is to construct for the second session three subtractive problems issued from the magnet situation. The second session is prepared as it has been described in 3.

V - Second session within the classroom (idem III)

VI - Final synthesis

Quite clearly, such a training progression requires a structure which fosters real collaboration between researchers, trainers, and teachers.

The contribution of research is discernible at two levels; firstly in the definition of training syllabus (whose choice and organization are directly linked to research into the elementary school) and secondly in the training itself during which the student-teacher is involved with his colleagues in research and intervention in live situations.

The involvement of elementary school trainers in this training is indispensable. This is because it is only on this condition that the student teachers can organize and set up their own learning situations: this shall be based on a priori introduction to experiment design methodology and situational observation and analysis.

An early assessment stemming from the trainers themselves shows that this sort of training course provides the future teachers with a clearer insight into the nature of the relation between the student-teacher and mathematics ehile, at the same time providing him with the tools of observation and analysis, too which, at a later stage, will be of use to him in class in order to question and improve his own teaching. Furthermore, an attitudinal change takes place on the part of the student-teacher. His interest is no longer centered morely on learner results (right or wrong) but on the underlying conceptions and the process by which they are attained.

What we are hoping for from a training course of this nature is that the future teacher will be more aware of the usefulness of inservice training, in close liaison with research and with the school, and that, at the same time, he will come to view his teaching more in terms of learner conceptions. Furthermore, we are hoping he will consequently be open to different approaches to the management of learning, not only in mathematics, but also in other fields.

NOTES

List of the team members involved with this training program

Bessot, A., Campa, C., Chevrot, C., Croquette, C., Eberhard, M., Guillerault, M., Neyret, R., Rival, G.

Word problems A and B

(A) Yesterday you were drawing or painting. I want to put your work and also some pictures on the blackboard.

I need 6 magnets to put on a painting; drawings are light, I need 4 magnets to put on one of them; pictures are lighter; one magnet is enough for a picture.

I've got 36 magnets. How many drawings, paintings, and pictures can I put on the blackboard?

(B) Your teacher has got 45 magnets. She wants to put sheets on the blackboard; they are two types of sheets: light ones and heavy ones.

She needs 4 magnets to put on a light one, 6 a heavy one.

How many sheets is she able to put on?

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RESEARCH ON THE ROLE OF THE FIRST OUT TEACHER OF MATHEMATICS AND THE IMPLICATIONS FOR PRESERVICE TRAINING

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BACKGROUND TO THE STUDY

During the last international congress (ICME4) Dunkley (1983) stated that "the gap between the ideal situation, as studied during preservice training, and the real situation in the schools has still to be bridged." More recently a survey of teachers in Britain in their first year of teaching (Department of Education and Science, 1982) found that in the initial stages of a teaching career, the match between qualifications and training and the work teachers were called upon to do in schools was of great importance. This theme was developed further in a "White Paper" (Department of Education and Science, 1983) devoted to "Teaching Quality", which attached a high priority to the fit between teachers' professional preparation and their subsequent tasks as one means of improving the quality of education. To ensure that mathematics graduates have the skills and knowledge to cope with their "first year out" in secondary schools, as well as providing the base on which to build and develop their future careers, the achievement of this match has been made a priority by the Diploma in Education team responsible for training mathematics teachers at Monash University.

It has been suggested (Johnston & Ryan, 1983) that there are few periods of time in the professional life span of teachers which will compare with the first year of teaching and Battersby (1982) recommended that teacher education institutions should give more emphasis to it in their programs. Others have questioned wither current courses effectively prepare students to be teachers adn in summarising their criticisms, Battersby pointed out that those responsible for preservice teacher education programs are seldom able to defend these "with anything more than opinions, impressions, hunches, and guesses" (Battersby, 1982).

The Cockcroft Report identified the problem of knowing what the newly qualified mathematics teacher should be equipped with on emerging from training and suggested it was essential that efforts should be made to achieve a consensus (Cockcroft, 1982). The key question is, however, whether this is possible and feasible through a program of research on the role and duties of new teachers.

In a study of beginning teachers, Otto, Gasson, and Jordan (1979) found that there appeared to be no available evidence of Australian preservice programs based on such analyses. In New Zealand, Battersby (1979) believed that trainee teachers would find programs both meaningful and relevant if they were derived from research on beginning teachers which had been obtained by their course designers. Hirst (1980) has suggested that there is widespread agreement that the Post Graduate Certificate in Education (PGCE) course in Britain should focus sharply, perhaps even exclusively, on the professional preparation of students for their first teaching appointments which in turn demands an accurate statement of the most likely duties during first teaching posts before detailed objectives and course content can be established. Koder (1983) in recognizing that it was neither possible nor desirable to provide, in the initial training period, all the knowledge, skills, and attitudes for a lifetime of teaching suggested that initial training should equip the prospective professional teacher with the fundamental skills associated with the educational task at hand.

PREVIOUS STUDIES

As part of a review of research on teacher education, Turner (1975) concluded that despite recent improvements in research in this field, the amount of dependable information available compared to the amount needed to formulate more effective policies and practices for teacher education was sparse. Subsequently Schalock (1983), in a discussion of research and development in teacher education, determined that nothing had changed to alter that conclusion. Johnston and Ryan (1983), in a review of research on beginning teachers carried out over 50 years, discovered that it was concerned primarily with attempts to improve the preservice curriculum. Despite this, they concluded that the research had made only limited contributions to the process of beginning to teach and on the initial training of teachers. Attention has also been given in other countries to research on new teachers in an attempt to develop strategies for teacher education curricula.

Teachers are often critical of their training in retrospect and in discussing the making of a professional mathematics teacher at ICME4, Rising stated that "if you ask a U.S. classroom teacher to describe his college preparation, he or she will almost with exception discredit all but what we call student teaching" (Rising, 1983). More recently in the USA, Joyce and Clift (1984) stated that "teachers believe that their training was (is) poor." Such general statements are, of course, limited in their usefulness in the context of improving training courses.

An analysis of the findings of almost a hundred studies found that there was an extensive, though somewhat shallow, description of professional problems encountered by first year out teachers (Johnston & Ryan, 1983). The most typical problems were generally in classroom management and discipline and their summary revealed that these, together with planning and organization, evaluation of students' work, motivation of students and adjustment to the teaching environment were perceived as the most common problems. A similar pattern was found in other studies (Dunkley, 1983; Griffin, 1983; Otto et al., 1979) with class control and handling constantly disruptive pupils the most frequently occurring problem reported by most teachers. In the area of teaching method, making the subject meaningful was the most common difficulty together with teaching groups with wide ability ranges and slow learners.

From the few findings on beginning mathematics teachers reported in the literature a similar pattern can be identified. The major problem perceived by a group of first year graduate mathematics teachers in a British study was discipline followed by teaching children of low ability and mixed ability groups (Cornelius, 1973). In response to a number of statements about the adequacy of their initial training, in a survey conducted for the Cockcroft Committee, a sample of first year teachers also felt that in general they had been prepared better for the subject content of their mathematics teaching and for classroom management and organization than for dealing with problems of discipline.

Those with the responsibility for training new generations of mathematics teachers regularly try to distill out of all the possibilities those essentials which, in the case of a post-graduate diploma course, must be achieved in under a year. In stating this, Blane and Clark (1983) also recognized that the value of a good match between the training and the task of mathematics teachers is a widely held aim that is not easily achieved. The task of establishing realistic goals for initial mathematics teacher education and the identification of possible strategies to achieve them was identified as an urgent priority for teacher education institutions in a report prepared for the Australian Association of Mathematics Teachers (1981). It recommended that support should be given to research in teacher education that identified effective practice. When commenting on the lack of research in mathematics teacher education during ICME4, Cooney stated that "the process of educating the professional mathematics teacher is too important to allow ourselves to be moved by whimsical forces" (Cooney, 1983). What is needed is firm empirical evidence on which to base course designs rather than opinions, impressions, hunches, and guesses.

In summary, there appears to be very little research reported in this field specifically related to first year out mathematics teachers. Most of the research findings arise from general populations of teachers with concerns mainly related to issues of classroom management and control, with only a few references to particular curricula and method areas. There are few recent report of attempts to use the research findings on first year out teachers to evaluate and improve preservice courses in a systematic way where they are shown to be deficient. This view is supported by Joyce and Clift (1984) who also reported that few teacher preparation institutions use research and development based innovations in teacher training. There seems to be agreement that follow-up studies are a useful means of assessing and improving programs of training but there are few examples of this in the literature, particularly for the training of mathematics teachers. Apart from the findings and recommendations of the Cockcroft Report there are few other recent relevant studies available. Cornelius (1973) looked at new graduate mathematics teachers in schools and Shuard (1973) conducted a pilot study of the expectations of heads of mathematics departments about new mathematics teachers in their schools. Both these studies have provided some information in Britain and Otto et al. (1979) have referred to some aspects of the professional life of teachers in Australia. Both the Shuard and Otto studies, together with the more general comments and findings of the Cockcroft Report have been influential in the design of this present investigation.

THE PRESENT STUDY

The study being carried out at Monash University is an attempt to redefine the objectives and subsequently the content of the mathematics education courses in the Diploma of Education. It was decided that for all the mathematics units in this post-graduate course for prospective secondary teachers, an attempt should be made to identify the realities of the situation into which they would be entering. To achieve this the sixty or so students who had graduate qualifications in mathematics and had completed the Diploma in 1982 were follows up through detailed questionnaires after their first two school terms of teaching. At the same time, the senior staff members in each of these first year out teachers' schools, responsible for their professional oversight, were also contacted together with a number of other teachers in schools in the State of Victoria, who carry out the teaching practice supervision of Monash mathematics education students.

The questionnaires were designed to provide details of what actually happened to these newly qualified mathematics teachers during their first year, both their problems and their expectations as well as the details of their work. Similar details were also elicited from their more senior colleagues. Specific comments were also invited from both groups about each item of the existing courses at Monash and their match with the realities of the situation within the wide range of local schools.

The questionnaire response during the first year of the project was good. From the 58 first year teachers who successfully completed the course, 46 replied. The response from the experienced teachers was also satisfactory and was particularly good from those teachers designated as the senior colleague of each of the first year out teachers. Both sets of respondents provided a wealth of information beyond the limits of this paper to describe in full. Much of it concerned aspects of the new teachers' levels of satisfaction with mathematics teaching as a profession, their proposed future study patterns and specific details such as teaching loads during their first year and other details of the programs carried out in their schools. For the purpose of this paper, a small representative selection has been made of some of the items which were of particular interest and influenced the way in which courses were planned for the following year.

Table 1

Frequency of Teaching Styles Claimed to be Used by First Year Teachers

Teaching Style	Often %	Sometimes %	Seldom %	Never %
Exposition	82.5	15.0	2.5	0.0
Discussion between teacher and pupils	60.0	35.0	2.5	2.5
Discussion between pupils	27.5	45.0	20.0	7.5
Appropriate practical work	25.0	45.0	25.0	5.0
Consolidation and practice of fundamental skills and routines	70.0	25.0	2.5	2.5
Problem solving, including applications of maths to investigational work	10.0	52.5	32.5	5.0
Outdoor work	2.5	7.5	50.0	40.0

Table 1 shows data on the frequency of the teaching styles claimed to be used by the sample of first year teachers. The question was prompted by Shuard (1973) and based in part on the suggestions made in the Cockcroft Report about what styles of teaching should be adopted for mathematics at all levels. It can be seen from the results that traditional methods still predominate in the local schools, even among our newly trained teachers.

Experienced teachers were asked to indicate the areas in which it was considered their newly trained colleagues were particularly well prepared and in which they were poorly prepared. The responses indicated that the first year out teachers were perceived to have a very good knowledge of all the mathematical content required for their duties and that their planning and work preparation was good. In terms of being "poorly prepared" the same experienced teachers observed difficulties with classroom management, discipline and setting of standards and to a lesser, but significant extent, catering for mixed abilities and individual differences. Also of concern was the apparent inability of many new teachers to provide mathematical explanations at appropriate levels for their pupils' understanding. The same concerns were also reflected again in a different questionnaire devoted to suggestions, additions, and improvements that should be made to the course content and these together with the whole range of the information provided from the questionnaires had clear implications for course design.

A previous study (Otto et al., 1979) revealed a number of major problems of beginning secondary teachers and a questionnaire based on the items from this earlier work was used as part of the present investigation. The detailed responses are not shown here but the newly qualified mathematics teachers reported that "Making the subject meaningful to pupils" and "Handling the constantly disrupting pupils" were their most serious and frequently occuring problems followed by a number of others also related to behavior and discipline. The findings from this group of teachers were virtually identical to those reported from the wider cross section of secondary teachers, from all curriculum areas, surveyed in the Otto study and also by other researchers in this field.

A final example of the results obtained from this survey relate to the current Basic and Further Mathematics Method course programs used in the Diploma in Education course. Both experienced and hew teachers were asked to indicate whether they rated each topic as "essential," "desirable," or "unnecessary." An an example, Table 2 shows the analysis for just those items related to Classroom Practice in the Basic course. They are listed in the rank order given as "essential" by the experienced teachers and close agreement can be seen between both groups. These results together with analyses of other categories such as curriculum, teaching round practice and other parts of the questionnaire were useful in guiding course planning for the following year.

Table 2

Extract from Table of Responses for Improving the
Basic Mathematics Course - Classroom Practice

	Experienced Teachers			First Year Teachers		
	"Essential"		"Unnecessary"	"Essential"		"Unnecessary"
	Rank	%	%	Rank	%	%
Class Management	1	97	0	1	87	2
Planning a Unit of Work	2	93	0	3	76	5
Worksheets, Tests, Assignments	3	87	0	2	78	0
Preparing a Package of Work	4	66	2	7	46	5
Exciting the Maths Student	5	56	4	4	72	0
Classroom Styles	6	50	0	5	54	3
Diagnostic and Remedial	7	50	5	9	41	5
Mini-lessons	8	47	8	6	51	6
Activity Workshop	9	29	5	8	46	11

DISCUSSION

A survey of research on the role of the first year out teacher reveals that, despite the existence of a consderable number of studies over the years, there are few accounts of these being used to develop or improve courses of teacher training. Few studies appear to have been devoted specifically to teachers of mathematics. A summary of the findings reveals a consensus that most of the problems of first year out teachers appear to relate to discipline and classroom control together with difficulties in teaching across the full range of ability. A similar pattern of findings has been observed in the Monash study for those particular areas and present no surprise. It seems, however, that if this type of research is limited to the "concerns" or "difficulties" identified by newly qualified teachers then the findings will always be fairly consistent, presenting no new ideas or implications for improving training courses. It would appear profitable to extend the research design beyond this level in an attempt to establish both the realities of life for new teachers, in terms of their expectations and the tasks asked of them, and also to elicit suggestions for improvements to courses. This project has attempted to do this and at the same time has enlisted the aid of experienced teachers to provide information.

The pattern of this investigation will be repeated for at least three years and the information will be used to review and update the course each year. At present, the study has reached the middle stage of this process, with data from the second year shortly available for analysis but there are indications that an improvement has already been made in the courses. A secondary, but important, by-product of this project has been the element of goodwill and sense of partnership generated between the teachers and the University staff, with benefit to both trainee and newly trained teachers.

Although the focus of this paper and the aim of the project has been towards mathematics teachers, the results indicate that the problems and realities for first year out mathematics teacher vary little from those of other newly trained teachers. It appears possible that one of the main reasons that so little of the earlier research has had an impact on preservice training is that the researchers and course designers and teachers have not had the necessary close relationship needed to effect an improvement. For this study, the research has been carried out by those responsible for designing and teaching the courses they are attempting to improve and this may be the key to an effective research design. Although it may not be possible to generalize the results to other situations and other training courses, it is believed that the pattern and techniques used here may have clear implications for those wishing to improve the training offered to their preservice students.

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THE ROLE OF THE HISTORY OF MATHEMATICS AND MATHEMATICS TEACHING IN TEACHER TRAINING

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There are many articles which emphasize the importance of the history of mathematics in the mathematics teaching. No teacher can introduce any topics in the history of mathematics without having the relevant knowledge about them. Therefore, it is quite necessary that the teacher training curriculum includes a course of the history of mathematics. However, what I shall report here is not concerned with how to introduce some topics in the history of mathematics directly in the mathematics classroom. It is the intention of this report to refer to some important ideas in the history of mathematics, to see how our precursors have made valuable efforts for realizing their teaching practices based on mathematical ideas and, by integrating them, how to formulate our present mathematics teaching. I wish to let our pre- and inservice teachers understand the implications of them. I shall give here some examples of them.

INTRODUCTION OF NATURAL NUMBERS

As is well known, the system of natural numbers was firstly axiomatized by G. Peano (1891). The most important primitive term ("key word") in this axiomatic system is "the next number," and the natural numbers defined by this system have the ordinal characteristics, not the cardinal. In 1905, the textbooks on arithmetic were firstly compiled by the Ministry of Education in Japan (there were no pupils' books of the first and second grades, but only teachers' manuals of these grades). The teachers' manual of the first grade shows us that the introduction of the natural numbers was completely based on the idea of Peano. It begins with writing and reading of natural numbers till 10, and then proceeds to reciting successive numbers correctly. Thereafter comes the addition of one-digit numbers whose sum is not more than 10. The sequence of dealing with such additions begins with 1 + 1, 1 + 2, 3 + 1, ..., 9 + 1, and then come 1 + 2, 2 + 2, 3 + 2, ..., and it ends with 1 + 9. Note that there were only sixteen years between the discovery of Pean and the publication of these textbooks. It shows us how Peano's achievement were mathematically epoch-making, but it also shows us that the new advance in mathematics reflected almost immediately on the teaching of mathematics.

It was not until 1935 when this treatment of natural numbers was improved. In this year, the textbooks were completely revised, and the pupils' books in the first and the second grades were published. The book of the first grade begins with the "ball-tossing," which is one of the most favorite mass-games for children. This activity also suggests us the learning of the one-to-one correspondence. In other words, here can be found the shift of emphasis point from the ordinal aspect of natural numbers to the cardinal one. Here we can find another example in which the academic achievement in mathematics reflected on the teaching of mathematics.

COMPOSITION AND DECOMPOSITION OF NATURAL NUMBERS TILL 10

The activity of one-to-one correspondence leads to the clear consciousness of the equivalence between two sets, but as yet, each number exists independently. The next step is to establish the relationships between natural numbers each other. Our elementary school teachers in twenties and thirties of this century emphasized that, after having established the correspondence between a set of objects and the corresponding numeral, it is essential to relate a number to the previously learned numbers less than it. By the aids of concrete materials, the teacher asks her pupils, for instance, that

2 and 3 is ?   5 is 2 and ?

The former is now called the composition of 5, and the latter is the decomposition of 5. Beginners often think the teaching of composition and decomposition as a preparation for the addition and the subtraction. Although it is partly true, the proper aim is to deepen the concept of number 5. This method is, of course, quite useful in doing the addition of one-digit numbers and the subtraction as its inverse operation. Let us take an example. When we add one-digit numbers, two cases occur. One of them is that their sum is not more than 10, and in this case, the only thing to do is reproduce the corresponding composition. If their sum is more than 10, then we preceed as follows:

> We want to add 7 and 4. Their sum is apparently more than 10. If we decompose 10 and ask ourselves "10 is 7 and ?", we obtain 3. Then if we decompose 4 and ask ourselves "4 is 3 and ?", we obtain 1. Therefore, 7 + 4 is 10 and 1, i.e., 11.

Inversely, if we do the subtraction 11 - 7, there are two methods. Both the them begin with the fact that 11 is 10 and 1 (this is the decimal number principle).

(the first method)	(the second method)
10 is decomposed into 7 and 3.	7 is decomposed into 1 and 6.
To compose 3 and 1 results in 4.	10 is decomposed into 6 and 4.
Therefore, 11 - 7 = 4.	Therefore, 11 - 7 = 4.

Needless to say, the pupil is not asked such verbal questions, but manipulates these processes by using the concrete materials. This method of teaching remains valid nowadays. We inherit the precious achievements which our precursors made more than sixty years ago. Consequently, we do not give our young pupils the addition table. They master the addition of one-digit numbers and its inverse through the repeated experiences with understandings.

MULTIPLICATION OF ONE-DIGIT NUMBERS

More than a decade ago, the archaeologists found the documents written on the pieces of wood at Dunhuang in North China. In one of them was found the list of multiplications of one-digit numbers. It is not expressed in the form of two-dimensional table, but in the form of ordered sequence. It is curious enough that the sequence starts with 9 x 9, and then follows 8 x 8, 8 x 9, and 7 x 7, ..., on the contrary to our usual order. It is said that in the more ancient document (about 1500 B.C.) the multilication list started with 9 x 9. The mathematical books were brought from China to Japan about the fifth century. Of course, we start it in the reverse order, i.e., 2 x 1, 2 x 2, 2 x 3, ..., but it remains unchanged how to write and read the multiplication facts in Japanese. The multiplication facts are called 'ku ku' because 'ku' in Japanese means nine. For instance, we say , and these Japanese words correspond to 'three', 'five' and 'fifteen' respectively. The pupil speaks it loudly and, by repeating it, learn it by heart. I shall add two comments here to avoid the misunderstanding. Firstly, is an elipitcal sentence written in Japanese. When the pupil writes the same fact in the Hindu-Arabic numerals, he always writes 3 x 5 = 15, and does not abbreviate the symbols x, =. Secondly, someone may be afraid that the pupil could not distinguish between addition facts and multiplication facts in his memory. There is no such case because, as I mentioned above, the pupil does not use the addition table, and does not speak addition facts loudly. It is only the multiplication facts that the pupil speaks loudly. It is usual in the Western countries that the pupil takes the multiplication table in the hand, and in doing the multiplication, he refers to this table. The ending result is the same, but how and where to start is different. By seeing or by speaking loudly. A few years ago, an educationalist asked me in astonishment, "Is there no 'ku ku' in the Western countries? Is it true?" I replied, "It is not exactly true. The difference lies in memorization method." Which method is more efficient pedagogically? I cannot give an answer, because it depends upon the social and cultural conditions, and also upon the personality of each pupil. Although it is sure in Japan that there are a few pupils in the upper grades, or even in the lower secondary, who cannot recite all of the multiplication facts correctly, almost all the pupils can do it in the lower grades.

A TRIAL OF EXTENDING THE SCOPE OF GEOMETRY BEYOND THE TRADITIONAL FRAME

Before the World War II, there was almost no elementary school geometry except for the names of simple figures and their perimeters, areas, and volumes in Japan as well as in other countries. The mathematics in the secondary school was divided rigidly into the algebra and the geometry, and the geometry was within the traditional Euclidean frame. Any preparation in the elementary school was believed to be not only useless, but harmful to the learning of the demonstrative geometry. The innovation movements of mathematics teaching at the beginning of this century gave our mathematics educators the strong influences, and our progressive precursors tried to provide the new course of mathematics, including geometry. But their efforts were fruitless at least in geometry. In 1941, just before Japan jumped in the Pacific War, the textbooks were revised, and the new course of geometry was adopted. The expectation of our precursors was realized. I shall show some examples in the lower grades in the elementary school. Figure 1 and 2 show the construction of varius patterns by the colored boards. The colored board is the isosceles right triangle with different colors on two faces. The children put two boards together along the longest sides to make a square. When they notice two specified boards, move one of them, and put it on the other, the children are conscious primitively of the concepts of translation, rotation, and reflection. Colors make a role of reflection clear as shown in Figure 1. Figure 2 shows two examples of tessellations composed of colored boards.

The reflection (or symmetry) is one of the most essential ideas in those textbooks. It is frequently used in various problems in geometry. It is asserted in the teachers' manual that the children recognize, for instance, an isosceles triangle as a symmetric triangle rather than a triangle which has two equal sides and two equal angles, and that the former is the intuitive recognition and the latter is the logical analysis. The children acquire the consciousness of symmetry by the observation of natural livings around them, but also by making various symmetric figures themselves. The paper folding makes it possible to make the such symmetric figures as butterflies and flowers. These activities lead to symmetric figures which have two axes of symmetry (see Figure 3). The paper folding is a favorite play for the children. Figure 4 shows how to make a box only by the paper folding. The children follow the indications given in the textbook to make the required box. It needs the spatial intuition to read the indications exactly. I think it was the authors' intention to make such an intuitive grasp of the space.

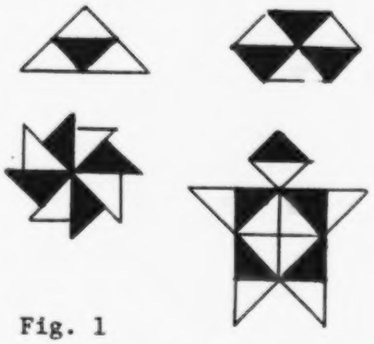
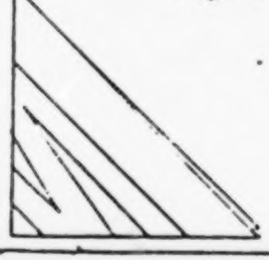


Fig. 1

Fig. 3


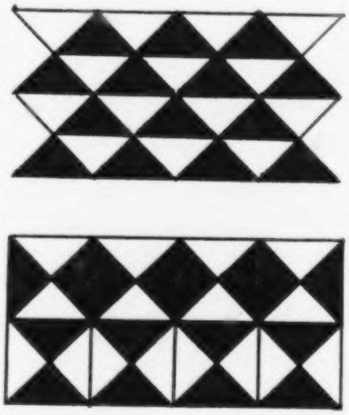


Fig. 2

The last example is mechanical curves. One of them is a spiral, and the other is an envelope (see Figure 5).

What was the philosophy on the geometry teaching? The teachers' manual explained it as follows. it is said that the intuitive recognition of the space has already been developed in quite young periods. It does not include in itself the analytical operations of thinking, but it is a very primitive grasp of geometric figures. However, the child grows up in the later periods to be able to think analytically and logically, he does not acquire the working recognition of the space, if his analytical and logical thinking does not accompany with his intuitive thinking. Moreover, the ability of intuitive recognition can be acquired more easily in the lower grades than in the upper grades and in the secondary school. This was their philosophy of geometry teaching. I must mention here that the philosophy and topics mentioned above were not inherited in the post-year periods. The geometry teaching returned back to the traditional framework. It is quite regrettable for us.

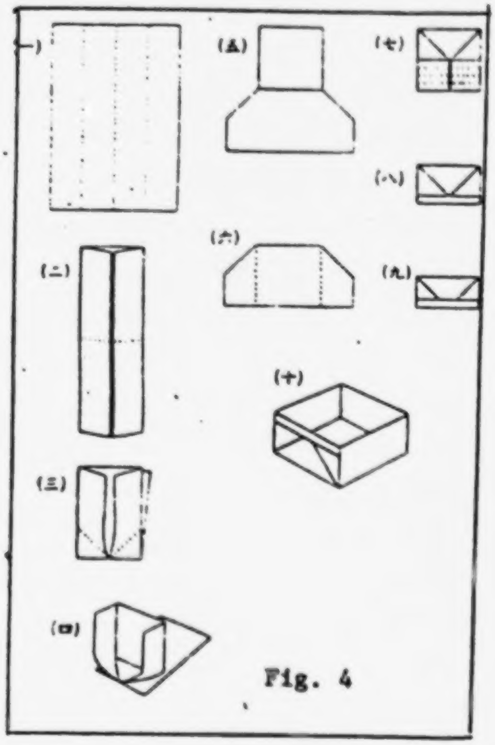


Fig. 4

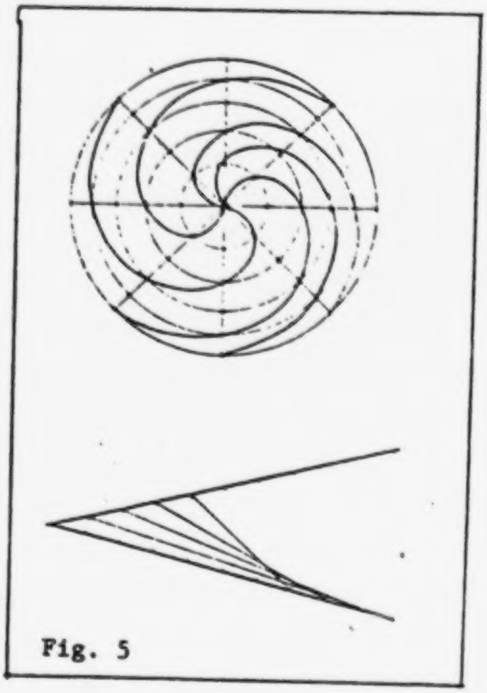


Fig. 5

BRINGING RESEARCH TO THE TEACHER THROUGH THE ANALYSIS OF CONCEPTS[1]

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INTEGRATING PEDAGOGICALLY RELEVANT RESEARCH

How soon can we teach counting-on in addition? What can a teacher do with the knowledge that most six-year-olds do not view 4 + 4 as equivalent to 7 + 1 and that later, when they do, they still might object to its symbolization 4 + 4 = 7 + 1? How come young children don't use subtraction to solve missing addend problems even when they are taught?

The solution of addition problems by counting-on from one of the terms (5 + 3 is five..., six, seven, eight) has been studied by many researchers. Fuson (1982) ha shown that this procedure involved a fairly sophisticated level of understanding. The word "five" in this count has four complementary meanings, all necessary for counting-on: it describes the cardinality of the first set, it anticipates the result and summarizes the act of counting this first set, it serves as a starting point in the enumeration of the second set. Carpenter and Moser (1982) have found that many children had to first learn counting-on in order to solve addition problems with sums greater than 10 in the absence of concrete objects. Groen and Resnick (1977) have taught addition to 4-1/2-year-olds on the basis of a counting-all procedure (counting from one) and they observed that eventually half the children spontaneously used a counting-on strategy. Thus, for an elementary school teacher aware of this research, two important questions come to mind: if counting-on can occur spontaneously, should it be taught? And if yes, how soon? These questions are far from trivial since teaching it prematurely may have surprising results as in the case of Valerie, a child in this age group who kne how to count from a given number. When addition the dots on two dice by counting all the dots, she was asked "Why don't you count by starting from the first die?" And she counted "5 (the first die), 6, 7, 8, 9 (the second die), ... 10, 11, 12, 13, 14" (the first die) explaining: "I did not count the dots on the first die" (Herscovics & Bergeron, 1982). The equivalence of sums is a notion which evolves quite later in the construction of addition. Piaget and Szeminska

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(1967) have shown that a five- or six-year-old believes that 7 + 1 is greater than 4 + 4 since he cannot perceive simultaneously the whole and its parts and thus focuses on the most evident parts (4 and 7). It is only later, around the age of 7 or 8, when he perceives simultaneously the whole and its parts, that he will accept the equivalence of sums by reasoning that they yield the same total, or by using addition, decomposition, and transitivity as in 4 + 4 = 8 = 7 + 1 (Herscovics & Bergeron, 1982). Piaget mentions yet a third form of reasoning, that of evening out the differences through compensation, as in 4 + 4 = (7 - 3) + (1 + 3). These results have an obvious implication for the teaching of the equivalence of sums, that of postponing it until the second or third grade.

Even at this later stage, many children will experience major difficulties with the symbolication of these equivalences. Indeed, several researchers (Behr et al., 1976; Kieran, 1980; Erlwanger & Belanger, 1983) have shown that the equal sign is perceived by many pupils as an "operator" symbol calling for an "answer". The fact that children maintain this initial meaning of the equal sign explains why they think that 8 = 7 + 1 is "all right but written backwards" and why they often "correct" 4 + 4 = 7 + 1 by crossing out the right hand side and replacing it by 8. Of course, a teacher aware of these cognitive problems might overcome them by temporarily introducing different symbols for decomposition and for equivalence of sums for instance 8 -> 7 + 1 and 4 + 4 = 7 + 1. Although many newer textbooks initiate the first-grader simultaneously to addition and subtraction in the hope that the two operations will then be perceived as inverses of each other, this objective may not be reached before grades 2 or 3. Steffe et al. (1983) have shown that six-year-olds who solved 7 + 5 = ? did not use their result to solve 7 + ? = 12 even when this last equation was written underneath the first one. Similarly, they solved 6 + ? = 10 by counting-on but did not use this result to handle 10 - 4 = ?, which they solved by counting back. A possible explanation might be found in the fact that addition is often taught on the basis of the union of two sets and subtraction as "taking away". But are these operations inverses of each other? When addition is defined by the cardinality of a terminal set resulting from the union of two disjoint sets, the inverse, which must undo the initial operation, calls for the separation of the terminal set into the two initial ones. On the other hand, since "taking away" is the inverse of "adding on" to an initial set, how can the child be expected to view it as the inverse of an operation based on the union of two sets?

While we have restricted ourselves to only one concept, early addition, we have shown that there are many studies which are pedagogically relevant. However, the existence of such pedagogically relevant research does not solve the problem of its integration by the teacher. Would the reading of research journals be profitable to teachers? Of course, we know that in general the answer to this question is negative. Research journals are not aimed at the teachers but at other researchers. The papers they publish report on research in a specialized language for the sake of brevity and precision. These papers emphasize research methodology and research results, rather than their pedagogical implications. And this is quite reasonable, since the objective here is to establish as thoroughly as possible a scientific basis for the conclusions reached.

It seems quite obvious that if we wish to bring research results to the teacher, we must report them in an appropriate language, stressing their implications for teaching, and publish them in journals aimed at teachers. But even when we manage to bring research to the teacher, does it solve the problem if integration? As we have shown in our introduction, most research studies deal with a particular aspect of a problem area. And rightly so, since to investigate a topic requires bringing it down to a size manageable for research. Moreover, any experimentation being run under specific conditions, its results cannot have teaching implications exceeding the particular constraints under which they were obtained. Thus, in general, research results have more a "local" relevance to teaching than a "global" relevance. And this is where lies the problem of integration, that of determining the relative importance of these local results in a more general context.

In order to assess the relative importance of research results pertaining to a specific concept, the teacher must have an overview of what is involved in the teaching and learning of this concept. For instance, without a general picture of early addition, he wouldn't know what to do with the research we have reported. Thus, bringing research to the teacher in any meaningful way involves a twofold problem: that of finding relevant studies, and that of helping the teacher construct a general framework in which the pedagogical implications of these studies could be integrated.

Finding a way of handling this double problem has been one of our main interests these last five years. In fact, it was at the heart of our research project appropriately titled, "The Integration of Research in the Training of Prospective and Practicing Teachers in Mathematics Education." We will now describe how we help the teacher elaborate a general frame of reference which makes him eprceive the child's learning of mathematics as a constructive process. Research results can then be used to clarify various aspects of these constructions.

OUR EXPERIENCE

For many years, teachers in Quebec have been encouraged by the Ministry of Education to keep improving their professional background. This has led many of them to enroll in various Certificate programs among which Certificates in the Teaching of Mathematics or Science and Mathematics. Courses in these programs last 12 to 15 weeks with three-hour weekly meetings in the evening. And it is with classes of about 30 elementary school teachers registered in these programs that we have been working.

The course we have designed deals primarily with the epistemological analysis of conceptual schemes and algorithms taught in early arithmetic, that is, number, the four operations, place value, the addition and subtraction algorithms. By epistemological analysis, we mean answering the question, "How does the child construct a given mathematical notion?" However, initially, such a question is far too complex for most teachers and a similar end is achieved by asking, "What does it mean to understand a given notion?" Of course, no two teachers will give identical responses and since in any of our classes we have teachers from grades 1 to 6, we receive an even greater variety of answers. In general, they are quite surprised by the diversity of their opinions. For example, to the question "What does it mean to understand addition of small numbers?", here are the kind of replies we get:

- It's adding a quantity to another one.
- It's regrouping objects; the student can say he has a lot.
- He can add to his card collection in order to have as many as his partner.
- He has memorized his sums.
- He can join and count from 1.
- It's learning by rote the mechanism of addition.
- In 5 + 3, he can count starting from 5.

- He can reproduce the equation starting from a drawing.
- He can represent an addition by a drawing, with objects of the same nature.
- He can make different arrangements.
- To join and count starting from the largest.
- It's the ability to explain that 2 and 3 equal 5.

While teachers may recognize in their answers different levels of understanding, they find it difficult to view them as different steps in the construction of addition. And yet, in the first three responses, we find the two fundamental ideas behind this operation: adding objects to an existing set, or joining two sets of objects. Even the preschooler shows evidence that he has a hold on these two action-schemes which constitute the preconcepts of arithmetic addition. Of course, even without counting, the child is aware that in either case, he ends up with more. These ideas are part of the child's informal knowledge of mathematics and are based on his experience which, of course, is not confined to school. They are a natural starting point for the construction of this concept and can be viewed as an intuitive understanding of addition.

These intuitive notions are essential in the construction of meaning for addition but are insufficient since, by themselves, they can only be used for some rough approximations. The two action-schemes must be coordinated with the counting process to yield addition in the arithmetic sense. At the beginning, the question, "How many do you have altogether?" sets off in the child the need to join or add the objects physically in order to concretely form a whole which he can then count. He thereby loses any concrete evidence of his initial sets. It is only later, when the two action-schemes can be performed mentally, without the need to gather the objects together, and when the child is familiar enough with the number word sequence so that he can start counting up from a given number, that he will be ready to use a more advanced procedure for addition, that of counting-on. Of course, systematically counting from the larger set is even more sophisticated since it presumes the commutativity of addition. These three counting procedures have all been mentioned by our teachers and constitute a major step in the construction of this concept. Their mastery and appropriate use is what we consider a procedural understanding of addition.

At the beginning, these counting procedures are used with sets of concrete objects. If the objects of one of the sets are hidden, the child can quite naturally use his fingers to represent them or use a counting-on procedure. If both sets are hidden, fingers can still be used as counters but this becomes increasingly difficult for sums exceeding 10. A gradual detachment from a concrete setting can only be achieved by a gradual memorization of sums. For example, a child remembering some numerical relationships can solve 7 + 8 as 7 + 7 + 1 or as 7 + 3 + 5. Only when all the sums up to 20 have been memorized can addition be freed from the necessity of a concrete representation as well as become independent of the counting procedures. Such a knowledge of addition, together with an understanding of place value notation, will allow for a meaningful learning of various algorithms for the addition of larger numbers. While memorization of sums is essential for the arithmetical development of the child, a premature emphasis on rote learning can be totally counter productive. Children can so concentrate on learning by rote that they might stop relating addition to the counting procedures and hence become incapable of handling unremembered sums.

Memorization of sums cannot be a substitute for an equally important activity, that of discovering fundamental properties such as the reversibility of addition, the equivalence of sums. For example, the pupil who knows that 7 + 7 is 14 and uses it to solve 7 + 8 as 7 + 7 + 1 indicates a fairly sophisticated knowledge of addition. In fact, to do this, he must perceive these additions as equivalent sums, which presumes a prior decomposition of 8 into 7 + 1. Furthermore, he also shows an awareness for the composition of addition, since he views the result of one operation as being equivalent to the result of two operations. This becomes increasingly important when later he is faced with handling strings of additions (2 + 5 + 8 + ...). Finally, he could also have solved 7 + 8 as 7 + 10 - 2 and this would have indicated a perception of subtraction as an inverse operation of addition since 8 is perceived here as 8 + 2 - 2.

The meaningful memorization of sums described above allows for the gradual detachment of addition from any concrete representation, as well as from any of the counting procedures. Such a detachment from the concrete is usually referred to as abstraction. But mathematical abstraction of addition, which constitutes a third level of understanding, involves much more: it involves the construction of invariants such as the equivalence of sums, which illustrates the invariance of the whole with respect to its parts; it involves the reversability of addition which implies decomposition (when addition is viewed as union) and subtraction (when addition is viewed as adding to); and it involves the composition of addition, that is the replacement of two consecutive operations by a single one.

We have managed to describe the first three levels of understanding early addition without any reference to mathematical symbolism. In a way, this shows that it is possible to reach fairly high levels of understanding using only enactive (actions) and iconic (images) representations. Of course, this does not mean that the introduction of mathematical notation should be deferred until the level of abstraction has been reached. In fact, we think that notation should follow closely on the heels of the addition procedures for then, an expression such as $2 + 3 = ?$ represents a problem associated with one of the addition action-schemes and the child can then solve his problem using any of the procedures he has just mastered.

The acquisition of meaningful mathematical symbolism is essential for the further mathematical development of the child. However, several studies have shown that the symbolic representation of mathematics creates specific cognitive problems. For instance, Ginsburg (1977) and Carpenter and Moser (1979) have reported that many children could handle simple arithmetic word problems involving addition and subtraction as long as they did not have to deal with them symbolically. The tendency of children to read from left to right and to perform their operations sequentially and one at a time, reported by Kieran (1979), may explain some of the difficulties experienced by children when working at the symbolic level and not encountered by them when other modes of representation are used.

Since mathematical notation brings about an increase in cognitive problems, one might be tempted to identify symbolization of addition as a fourth level of understanding. But the work of Erlwanger (1973, 1975) on individualized programmed instruction has shown the extent to which students could succeed on some tests by manipulating symbols which were devoid of any meaning to them and by basing themselves solely on the disposition of the symbols to derive idiosyncratic rules. We thus must conclude that, by itself, the correct maniuplation of symbols cannot be taken as a criterion of understanding. This has led us to consider symbolization as relevant to a fourth level of understanding only if prior abstraction of the concept has occurred to some degree. We have called this fourth level of understanding the formalization of addition.

These last pages provide a summary of how the child may possibly construct addition over a period of two or three years. A teacher who has participated with us in such an epistemological analysis has an overview of what is involved in the learning of this concept. Thus, we can now truly appreciate the contributions provided by relevant research. having to decide how and when to teach counting-on in addition, such a teacher would be most interested in the results obtained by Secada, Fuson, and Hall (1982). These researchers have identified three subskills leading to counting-on: (1) being able to count up from a given number; (2) recognizing that the number of objects in the first set will correspond to the last counted object in that set, and (3) recognizing that the first number word used in counting the second set is the one following the last number word used in the first set. They have shown that first-graders could be successfully trained on subskills 2 and 3 in one session, and that 88% of these then spontaneously counted-on in addition.

BY WAY OF CONCLUSION

We hope to have shown, using the notation of early addition, that there exists a significant amount of research which is pedagogically relevant. While by the very nature of research, individual findings are "local" rather than global, their relative importance can only be appreciated in the context of a more general frame of reference providing an overview of the processes involved in the construction of a given concept. And it is precisely the role of epistemological analysis to provide the teacher with such an overview. He can then, not only appreciate the research results, but also use them effectively in his teaching.

As we have pointed out, the epistemological analysis of a concept is a task too difficult to expect from untrained teachers. Thus, we have tried to bypass this problem by raising the question of understanding. In this way we were able to identify four levels of understanding of early addition which can be viewed as four stages in the construction of this notion. The epistemological analysis of other mathematical concepts has shown that these same levels of understanding could also be found in their construction. This has led us to search for criteria which might enable us to characterize these different levels of understanding. And it is the sum total of these criteria that constitute what we have called a Model of Understanding (Herscovics &

Bergeron, 1983).

If initially, analyzing the construction of mathematical concepts was at the foundation of our Model of Understanding, on the other hand, this model is now a useful tool in training teachers to try their hand at epistemological analysis. For indeed, given a mathematical concept, they now can ask "What can we find in the child's experience and actions that might be considered as intuitive understanding?" They can then try to relate it to the arithmetical procedures they teach. And they no longer stop there for they now may ask "What would constitute a mathematical abstraction of this notion?" This would insure that understanding a concept is not mistaken for the correct but meaningless manipulation of symbols. By answering these questions, the teacher achieves an epistemological analysis. Such analyses induce a constructivist approach to the learning and teaching of mathematics and provide a framework for the integration of research results.

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TEACHERS AS RESEARCHERS

THE ROLE OF THE TEACHER IN THE RESEARCH ENTERPRISE

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I chose the above title because there seems to be increased interest in the role teachers play in the research enterprise and in discussing issues inherent in defining that role. If the title were case as a question, a glib answer would be a listing including such categories as subject, informant, participant, and colleague. I would like to consider not a list but rather various perspectives of research and implications of those perspectives for defining the roles researchers and teachers play.

RESEARCHERS AND TEACHERS AND THE ROLES THEY PLAY

In discussing the roles of teachers and students with respect to motivation, Dewey raises the question as to whether the teacher's job is to provide motivation for whatever he wishes to teach or whether the student has the responsibility to exert effort to understand what may not be interesting at all. With a slight twise, I pose the following analogous consideration.

> Is it the job of the researcher to provide results for whatever it is he wishes to study, or rather, it is the job of the teacher to exert effort to understand what may not be relevant at all?

As Dewey suggests in his discussion, the issue is not one of resolving the question so much as unearthing an assumption that is problematic, that is, the assumption that what is to be studied, the object of the research, is, in a sense, "external" to the teacher. Analogous to Dewey's discussion about teachers and students, the question presupposes a separation between researcher and teacher, a separation that can take several forms. It suggests a different agenda, e.g., the researcher being interested in "Why" and the teacher being concerned with "How". It increases the likelihood that the questions asked flow from the researcher and possibly a theoretical perspective rather than being grounded in the teacher's contextual framework. it defines the role of the researcher to be the questioner, the observer while the role of the teacher is to provide answers and a basis for observations.

Mitroff and Killman (1978) discuss four types of approaches to social science research. As personified, they are: (1) the analytic scientist who values objectivity and precision, embraces the constructs of reliability, external validity, rigor, and tries to maintain an "acceptable" distance between himself and the "object" being studied to insure objectivity, (2) the conceptual theorist who seeks multiple explanations and who attempts to resolve apparently conflicting explanations, (3) the conceptual humanist who acceps Comte's statement that "Humanity is alone real; the individual is an abstraction," and who defines a problem by reference to the concept of one's personal being, and (4) the particular humanist who emphasizes the uniqueness of individuals and who often relies on the utilization of case studies.

The analytic scientist is the most likely to emphasize separation between researcher and teacher as an emphasis is placed on atomization and the importance of precision in defining variables. The humanistic methodologies tend to embrace more holistic approaches with emphasis on teachers' individuality. While the researcher still may have an "external" orientation to the teacher, he attends to understandings and meanings, i.e., conceptions, the individual holds, conceptions that are necessarily internal to the teacher. Here the researcher must be closer to the teacher for otherwise the validity of the meanings ascribed to the teacher would be seriously questionned. The power of the humanistic methodologies lies in the richness of the meanings revealed and in the ability of the researcher to interpret the actions of the teacher given the teacher's contextual and conceptual framework.

Action research is perhaps the ultimate of methodologies that reject the externality assumption. As described by Kemmis (1982) it is a method that involves self-reflection, a commitment to self-improvement and a participatory perspective between researcher and teacher even to the extent that the distinction between the two becomes blurred. Action research requires, according to Kemmis, a rejection of the positivist notion of rationality and absoluteness of truth. As such, it is distinguished from the analytic methodologies and the implicit assumption that reality is something to be discovered, i.e., something external to the individual. Action research is inherently humanistic in nature as it deals with issues internal to individuals and their working conditions.

Still another view of the roles researchers and teachers play is given by Cobb and Steffe (1983) who discuss the importance of the researcher as a teacher and modeler from a constructivist perspective. Constructivism implies that knowledge is idiosyncratic, specific to individuals; hence the researcher's task is to understand how the individual "arrived" at his meanings. Constructivism does not hold to the assumption that knowledge is truth in some external and absolute sense. The constructivist teacher is interested in understanding how the student constructed his mathematical ideas and uses that understanding to determine his teaching activities. The constructivist researcher is interested in how the teacher derives meanings from classroom events and tries to understand the teacher's "world view" in an effort to understand the nature and origin of teachers' instructional decisions. The issue is not what knowledge is needed to become a better teacher, but rather what is the nature of the knowledge held about students' understanding of mathematics.

A pair of related articles by Wheeler (1970) and Bishop (1971) highlight several issues of interest here, in particular, the notion of a scientific teacher. Wheeler states that the objective of science is truth and that the method of science is observation. Subsequently he argues that what is needed is scientific teaching, i.e., teaching which portrays the teacher as an observer of students rather than a presenter of mathematics. The scientific teacher

> will start with the tasks to be done and will consider how the attention of the children can be focused on them. He will consciously withdraw as much of himself as possible so that he will not be an interference to the activity he wants to promote, but to the tasks in hand, he can be an impartial observer of their actions as they tackle them." (Wheeler, 1975, p. 25)

Wheeler's notion of scientific teaching places an emphasis on reflection and introspection as the teacher is asked to monitor more than present. The scientific orientation is internal to the teacher. While questions about scientific teaching can be asked from an external vantage point, it is more likely that meaningful progress would stem from collaboration rather than separation.

Bishop's notion of a research-oriented teacher has a slightly different twist. According to Bishop, such a teacher has the following attributes:

1. Having an awareness of what research has done and constructively criticising that research.
2. Having an understanding of the role of research.
3. Having the ability to apply the results of research.
4. Having the ability to look objectively at his teaching.
5. Having continual attacks of 'curiosity.'

The first three attributes involve knowledge that originated outside the teacher's experiences whereas the latter two involve actions internal to the teacher.

Wheeler's notion of a scientific teacher focuses on a conception of teaching that has to do with the process of teaching and how that process dictates relationships with studens. It is an "inward" looking perspective in that it obligates the teacher to reflect on her role in defining teacher/student relationships. Hence, it makes little sense for the researcher to distance himself/herself from the (scientific) teacher. Bishop, on the other hand, reveals two different roles for the teacher, one being a consumer of research findings and the other, as emphasized in points 4 and 5, being an individual who engages in reflection and introspection. The value of research for the "consumer" teacher is more likely to be judged by its relevance and utility. But for the latter individual, research has a different orientation. Research is not something that happens "out there" in which case, it makes sense to ask about its relevance and utility but rather it is something that is an integral part of the teacher's conceptual makeup. In short, the externality question becomes moot.

THE ISSUES OF GENERALITY AND PROGRESS

Lakoff and Johnson (1980) emphasize the importance metaphors play in trying to understand human thinking and behavior. Elsewhere in this Congress, I have argued that metaphors also pervade our professional lives, i.e., the way we conceptualize and study problems. The engineering metaphor, for example, emphasizes a search for general truths and principles that can help guide our teaching. If one accepts such a metaphor, then progress in educational research is determined by the "power" of the generalizations to yield pedictions and reveal useful prescriptions. Generality is the key for the goal is to have a sufficient knowledge base that most or at least many educational problems can be addressed substantively and successfully. Analytic methodologies (Mitroff & Killman, 1978) are an integral part of such an orientation. The externality assumption is accepted as the researcher positions himself as the controller of the research activities with all the precision that can be mustered.

But less technical metaphors suggest another kind of progress, one in which teacher's individual conceptions are the primary focus of attention. Here we have the notion of naturalistic generalizations (Stake, 1978) that are derived from tacit knowledge, knowledge which is a composite of shared meanings, experiences and emotions among humankind. Naturalistic generalizations and their inherent meanings provide the basis for communication among lovers, athletes, mothers, and fathers despite the absence of a common spoken language. STake has argued that to generalize in a "natural way" is to be both intuitive and empirical. As Eisner (1981) put it, there is generality in the particular. Blake said it so beautifully in the following way.

> To hold the world in a grain of sand
> And heaven in a wild flower;
> To hold infinity in the palm of your hand,
> And Eternity in an hour.

Our concept of generality and how we view progress is central to how we view the role of the teacher in the research enterprise. Generality in the sense of absoluteness suggests analytic methodologies with their emphasis on objectivity, impartiality, and attempts to discount biases by whatever means are available. The teacher is an entity to be observed, and assumed to be representative of a larger population. The externality assumption is accepted. But if the view is accepted that generality is also a characteristics of particulars, as for example, in the case of Benny (Erlwanger, 1973), then quite a different perspective emerges. Humanistic methodologies, action research and the notion of introspection and reflection, necessarily idiosyncratic processes, can serve as a basis for understanding how teachers grow and change professionally, why they make the decisions they do given the environment in which they exist, and what impediments exist for making innovations. Such understanding can form a basis for meaningful naturalistic generalizations and interpretations, notable progress to be sure. Here the teacher plays a different role, one defined as a collaborator rather than as a subject who is somehow representative of a larger sample. It is through collaboration that the researcher can come to understand the phemonology of teaching.

If one views himself as an analytic scientist with all of its implicit meanings and believes in the importance of generating statistically reliable principles of teaching, then one is obligated to distance himself from the teacher and to define the teacher's role accordingly so as to insure objectivity. But if one accepts progress in the form of naturalistic generalizations and desires to understand meanings teachers ascribe to classroom events, then the researcher and teacher will play quite a different role, one more akin to collaboration in which separation is minimized and subjectivity plays a prominent role.

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THE CARRYOVER FROM COLLEGE TO THE SCHOOL OF MATHEMATICS EDUCATION THEORY

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Colleges and universities are entrusted with the task of producing trained graduates for schools. That teacher training courses are still much as they were in the past (various combinations of foundation, curriculum and practice-teaching units) might suggest that the clients are satisfied with the product. Reports from many sources, however, do not tend to agree with this assessment. In fact, the common element of many reports is that of dissatisfaction in some form with the teacher training process. The most common concern is that teacher training courses are not "relevant" to the school context where the graduates must eventually perform their tasks as teachers. It is not always clear as to what it is to be relevant in this situation because by its very existence, the college cannot simulate the school and must, therefore, be seen as different. In an article which is most critical of teacher training in the colleges, Spillane (1976) claimed that training should occur in schools wehre children are because colleges have failed to produce successful graduates. Quoting a National Study Commission on Undergraduate Education and the Education of Teachers, Spillane reports that "... Almost two-thirds of the senior teacher education candidates agreed with the statement: 'Most American colleges reward conformity and crush creativity'...96% of senior teacher candidates said coure work should be more relevant" (p. 435).

Spillane would like to break the hold that colleges and departments of education have over the training of teachers. For him, teacher organizations and local school boards are important in the training phase and should be part of the course planning and implementation process. I am sure that many colleges would argue strenously that they do involve their students in real schools as part of the practicum and so claims of ireelevance are baseless. If these school experiences are, however, controlled in the main by the college sector and form only a small part of the total course time, then such criticisms still ring true. For many students, school experiences provide little more than a confusing and frustrating conflict between what is expected by the competing interests of school and college supervising groups.

This need for a closer connection between the school/teaching interests and the college/central administration sections is echoed by Buckley (1980). his comparison of a group, when students at college, and later as first year teachers, indicated that problems of the beginning teacher far outweighed satisfactions. Most of these problems were "school-based, prescriptive and poor curricula, lack of facilities, testing procedures, unfamiliar clerical tasks, inaccessible resources...." (p. 78) Here the faults are spread between school and college, but the unsatisfactory transition from college to school is clear. Surely an easier progression must be possible.

Other problems are noticed when the preservice trainee is investigated. Campbell and Wheatley (1983) identified three stages of development through which student teachers passed. They labeled these

"1. concern with self
2. concern with teaching actions and student's behavior, and
3. concern with learning." (p. 60)

and suggested that some students would not reach the final level before they had completed their training. Such students would surely be less sensitive to the subtleties of the teaching/learning environment met in their first school.

Corcoran (1981) has also investigated the transition phase and has identified one element common to many first year teachers. She has called this factor the condition of not knowing and describes it as

> "What complicates this inevitable shock of not knowing for the beginning teacher is the need to appear competent and confident. Even though one is a beginner, one is also a teacher. Implicit in the role of the teacher is the notion that contradicts the very essence of being a beginner." (p. 20)

Lamme and Ross (1981) pose the further question whether college courses influence the teaching style of the students in any significant way. Though Lamme and Ross were interested in the effect of inservice courses in practicing teachers, the same question has been asked to preservice courses by Tabachnick (1980) and McCaleb (1979). Lamme and Ross concluded that teachers found too many constraints which "tended to reinforce traditional models of teaching and to discourage any deviations." (p. 29)

Smyth (1982) in advocating a clinical supervision approach, also discusses the "glaring disparity between their (teachers) intentions and their actions." (p. 134)

These examples suggest that influences (successful or otherwise) on the preservice teacher are quite varied. Taylor (1980) has suggested a model to explain these competing influences in his use of an ecological press paradigm, as shown in the figure below.

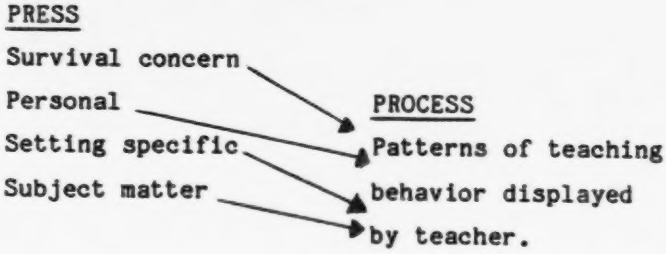

Figure 1.

The survival concerns have been well documented by Fuller and Brown (1975); the personal press relates to the student's predilections towards teaching strategies and perception of their college preparation; the setting specific press results from the school setting and the teachers involved in that environment and the subject matter press that which is demanded by the unique character of the particular content area.

For Taylor, teacher educators must acknowledge these constraints which tend to make life for the preservice and first year teacher so complicated. That students may tend to revert to styles which can be found in the teaching they last experienced as children in primary grades, that school-college conflicts produce, at times, conflicting messages, that different content areas may demand different behaviors from teachers, are all quite true. How courses in teacher education should take account of these is less certain.

ELEMENTARY MATHEMATICS TEACHERS AND THEIR TRAINING

In an attempt to find out what actually took place in classrooms, Price, Kelly, and Kelly (1977) designed a survey of the classroom practices of elementary school teachers across America. Their aim was to find out if the apparent changes to mathematics education (well documented in mathematics education journals and extolled by lecturers in pre- and inservice courses) were recognizable in teh actions and beliefs of teachers. Their summary was that

> "The overwhelming conclusion to be drawn from these findings is that mathematics teachers and classrooms have changed far less in the past fifteen years than had been supposed.... Teachers are essentially teaching the same way that they were taught in school.

> Almost none of the concepts, methods, or big ideas of modern mathematics programs have appeared in this median classroom." (p. 330)

A further interesting conclusion they make is

> "If there are indeed declines in mathematics test scores, only a small decline can be attributed to 'new mathematics' since little 'new mathematics' has actually been implemented in the classroom." (p. 330)

A survey of schools which provides class experience for the author's students bears out this analysis. Though there are individual teachers and schools exhibiting interesting, innovative and sound approaches to the teaching of mathematics, there are far more reports of classes in which the teacher uses few (if any) illustrative or manipulative materials presents the mathematics in a lecture-expository style to the children who then practice the material in whole class groups. Student-teacher dialogue is restricted to the repetition of "correct" answers supplied by the teacher in some prior session. Though most schools have a school-curriculum, either developed by a staff group or taken out from some other source, individual teachers too often teach in isolation from the rest of the school. Whilst it is recognized that, by and large, most classes mesh with their neighbors in a satisfactory sequence, there are, at times, quite important mismatches and inconsistencies which mitigate against a most efficient climate for successful ongoing mathematics learing.

Davis (1979) summarized the mathematics reforms of the 1960s and 70s in the following way.

> "A number of worthwhile innovative curriculum improvements were developed in the U.S. Their impact has been somewhere between slight and insignificant." (p. 162)

In commenting on this, and other issues relating to maths education in the U.S., Keitel (1980) concluded that

> "The U.S. has decided to afford the luxury of spending enormous amounts of labor and expense for design of reforms but has left the outcome to a more or less accidental acceptance by, first, the school districts, and secondly, the teachers and students." (p. 124)

Matthews (1981) had similar observations to make when investigating the poor results of a group of apprentices.

> "Without exception, lessons usually followed the pattern of work being explained by the teacher on the blackboard and then exercises

> being set from the blackboard, textbook, or hand-out. On the whole, no practical work at all was included in the lessons. Many of the apprentices reported that their teachers covered the work at a rate suited to the most able in the class which invariably did not include them. Consequently, they were left further and further behind, being unable to 'catch-up'. (p. 26)

That the practice of mathematics education fails to live up to the theory is surely a discouraging comment on schools and on the preparation of teachers. Many of the problems associated with teacher training in general, as outlined in the first part, apply equally to the training of mathematics teachers. There are, however, some unique problems which arise in the training of mathematics teachers at the elementary school level.

Mathematics anxiety has become a recognized, if little understood, phenomenon in the last decade. As the students who are likely to enroll in primary school training courses are unlikely to have taken many specialist mathematics courses or have had a lack of success with secondary courses, then these same students more often exhibit anxiety towards mathematics. Courses which are designed to prepare them for teaching have a three fold purpose.

a. to ensure that the students are aware of and competent in the content which they are to teach.

b. to develop skills in planning, implementing, and evaluating units of work in mathematics.

c. to encourage and foster positive attitudes in students towards mathematics.

It is not easy to design courses to meet these three demanding needs. It is no wonder that some students are unable to reach suitable levels of developing in all three areas before completing preservice training.

Perhaps an even worse unsatisfactory combination would be to find students whose one level of mathematical understanding was at an unacceptably low level. In an investigation of the mathematical competencies of primary teachers in Victoria, Hind (1981) concluded that his data "indicated that many teachers have not mastered the content of the primary mathematics curriculum." (p. 310)

Hind quite correctly refused to blame any group for these poor results, preferring to see teachers as the product of a societal press which at times turns into a cycle of neglect. Teachers with less than satisfactory maths experiences teach in ways which present less than adequate mathematical experiences to groups of children who eventually become teachers, etc.

Freudenthal (1977) argues quite eloquently for an acceptance of this state of affairs and for a positive approach to the training of such preservice students. He says

> "There are countries where future primary school teachers must learn a lot of mathematics they will never use.... At our training institutions for primary school teachers, the majority of students did not take mathematics...beyond the ninth grade.... I must confess I consider it as a gain that we cannot teach these teacher students highbrow mathematics. We must be satisfied to teach them the same mathematics we teach primary school children, the same subject matter, albeit at a higher level of understanding. I think this is the secret key to what I called integration of subject matter with its didactics, not only at the primary levels, but at any level." (p. 373)

Freudenthal to me is suggesting that content, method, and attitudes can all be covered if the integration of all three components is carefully planned and thought through at the level of the students entering teacher training courses.

MATHEMATICS EDUCATION THEORY

At a recent conference of mathematics teachers and educators, a panel of prominent researchers was asked to list what they accepted as the basic tenets of a mathematics education theory. At the time, I was surprised that these eminent researchers were not willing to state anything more than self evident truisms. All else was still too tentative and uncertain. The absence of a formalized theory of mathematics education has since been reported by more and more workers in this field. The need for such a theory and the signs that sensible starting points exist also receive constant recognition. With such a theoretical basis, mathematics educators will be more able to construct the kind of training course to generate capable and competent teachers at mathematics. How much progress has been made since Shulman (1970) wrote his excellent article which attempted to chart a path through the apparent conflicting theoretical statements?

Perhaps it is important to recall Shulman's cautionary note about the trap of attempting to compare contrasting and possibly quite unrelated theoretical positions. Too much time is spent justifying position A against position B. Theoreticians may well benefit from this esoteric debate, but what are the benefits for the practitioner? Similar comments on such conflicts are made by Keitel (1982) in her comparison of mathematics education in the USA and USSR.

> "The development of mathematical education in the US faces a dilemma which results from the existence of two antagonistic schools of learning psychology, the behaviorist approach of a Piagetian type on the other hand.... Moreover, their specific strengths and weaknesses are connected to opposing socio-political views of education: behaviorism emphasizing skill training and utility, and cognitive development an individual-centered view of the basic goals of education." (p. 117)

It is interesting to note that Keitel finds similar dichotomies in the supposed homogeneity of the Soviet system.

In the absence of a legitimized theory of mathematics education, how is the mathematics educator to proceed? There are three possible directions to take:

1. do nothing and wait for a well defined theory of mathematics education to emerge.
2. take sides with one or the established learning theories and prepare to do battle.
3. create a "workable synthesis" of the most sensible ideas which can be culled from the competing theoretical positions available.

To the author, the third position remains the only defensible position if one recognizes the role of mathematics educators in the school system. Perhaps in time, discussion and debaste about these "workable syntheses" may well lead closer to a theory of mathematics education. Remarks by Gagne (1982) in a recent article on the psychology of mathematics education serve to highlight these issues. His discussion of the nature of the terms concrete and abstract as they relate to mathematical content seems to create conflict for anyone who would espouse a Piagetian model. However, when analyzed closely, his position does provide the practitioner with many sensible ideas for putting into action ideas which would not necessarily contradict those which might flow from an analysis of an alternate theoretical viewpoint.

It is this desire to build something from the theories rather than wait till a definitive winner emerged which has led to the development of the pilot project described below.

TEACHERS' PERCEPTIONS OF MATHEMATICS EDUCATION THEORY

Students at the Darling Downs Institute who are enrolled in primry preservice training courses study two compulsory units in Mathematics Education. These two units combine content and method strands in an integrated structure. The course aims to produce students who

- can operate mathematically with the content of the primary school programme.

- understand the major issues relating to children's learning of mathematics.

- develop experience in organizing, implementing, and evaluating units of work in mathematics.

- possess positive attitudes to mathematics and its teaching.

The dichotomous demands imposed by schools and colleges have been recognized and the units have been created to stand on their own as units which will prepare students for the school experiences as teachers of mathematics in primary schools. There has been an attempt to translate into practical terms the theory which surrounds mathematics education. Lecturers involved in the units are aware of the task involved in convincing the students that they should accept the theoretical base as well as practice methods and develop teaching hints. Yet even if the lecturing team is most successful in convincing the students of the merits of the theoretical assumptions underpinning the mathematics education courses, it is a further task to establish these base concepts so securely that the students will maintain their convictions when they commence teaching. Past mathematical experiences and the models provided by teachers in local schools tend to make this a difficult task. Will the pressures exerted by colleagues, the problems associated with beginning teaching, the difficulties involved in awakening in their pupils an interest and a desire to become mathematicians all erode the strong base?

A pilot project has been commenced at the Darling Downs Institute which will trace students through their first few years of teaching. The project, though simple in its concept, has far reaching implications for the three areas outlined in the body of this paper. An instrument is being developed to establish the importance which students/teachers attach to statements which relate to assumptions underlying the courses they studied in mathematics education during preservice training. The students will be asked to respond to the position statements at the end of their training at college. They will again be asked to respond to the test instrument during their first year of teaching. The teachers will be asked for reasons for any changes in responses. Through interview and questionnaire, the teachers' progress will be mapped and the factors which contributed to these changes catalogued.

The project is expected to evolve in the following directions. Initial responses to the trial questionnaire will help clarify the statements relating to the theoretical base of mathematics education courses at this college. Responses from the teachers will help to detail the nature of the college/school interaction. The degree of support and the areas of conflict should be established as the teacher's progress is mapped. The strength of the teachers' convictions will provide information about the design of the courses and the applicability of their assumed theoretical bases. In this formative stage, with little hard data to produce for any kind of meaningful analysis, it is perhaps of value to describe a significant direction which the reading of the literature has imparted to this project. Ainsberg and Newman (1981) point out that

> "It is necessary to treat as problematic whether any message (reproductive or transformative) sent by a teacher education program is received by a preservice teacher as it was sent or whether some distortion or even inversion occurs." (p. 7)

Rogers and Schuttenberg (1979) make a plea for colleges/universities to tidy up their own practices "before seeking further to assist students in developing the competencies essential to coping with, deviating from and/or altering the norms of the elementary and secondary schools in which they will work." (pp. 39-40)

Zeichner and Tabachnick (1981), in discussing the notion that school experience "washes out" the liberalizing influences of teacher education courses, say that "by focusing on how things were to be done without asking students to consider WHAT was to be done and WHY, the university initiated discussions which tended to encourage acquiesence and conformity to existing school routines" and "We can no longer assume that the role of the university is necessarily a liberalizing one and that the schools are the only villains in the creation of undesirable teaching perspectives." (p. 10) Teacher training institutions must therefore become more familiar with the complexity of the issues and the variety of the constraints impacting preservice mathematics education students and design courses in the light of such information.

If it is concluded that quite radical changes are required in the ways in which mathematics is presented to children in schools, then the training at both preservice and inservice levels should be ideologically conceived and planned in the political context which it must inevitably accept if it is to be successful. The ultimate goal of the project commenced at the Darling

Downs Institute is to define the theoretical bases of the courses offered in mathematics education so that their influence on the students who undertake their study is powerful enough to effect changes when the students commence teaching.

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CLASSROOM IMPLICATIONS OF RECENT RESEARCH ON RATIONAL NUMBERS

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Any discussion of changes in elementary school mathematics curriculum is certain to contain arguments concerning the place of common and decimal fractions. With the availability of calculators and the increasing use of the metric system, especially in the English-speaking world, there is a need for earlier exposure to decimal notation. This need, coupled with the difficulties students have experienced particularly with operations on common fractions, would imply for some a corresponding delay in fraction operations. The purpose of this paper is not to reiterate these arguments but to look at a sampling of the research related to rational number learning. Conclusions of this research from a number of sectors are very consistent. There is a serious lack of understanding of rational numbers. What can be done about it?

TOWARD A COMPLETE UNDERSTANDING

Kieren (1980) has described four (or five) subconstructs or experience bases for rational numbers: measure, quotient, ratio, operator (and part-whole). He argues that the first four subcontracts are substantive and that the fifth may not lead to an adequate understanding of rational numbers.

The measure subconstruct is evident when we want to quantify the surface area of a region or the length of a segment. A suitable unit is chosen and fractional parts are derived by successive partitionings to make the measurement more precise. The measure may be expressed as both a common fraction and as a decimal fraction whenever the partitioning is standardized to tenths or hundredths, for example. The formal meaning of quotient numbers is rational numbers which are solutions to equations of the form $ax=b$ where a and b are integers and $a=0$. In another sense, quotient numbers are rooted in experiences such as sharing three chocolate bars among five persons. A third subconstruct of rational numbers is ratio numbers. "This idea manifests itself, for example, when one looks at a one-to-three mixture of flour and water and observes that one-fourth is flour" (Kieren, 1980, p. 72). The fourth and final bona fied subconstruct is operator or mapping numbers. One example given is that of a "1/8 operator" which can be a mathematical model for a machine that packs gum eight sticks to the pack. The number of sticks is paired with a number of packs one-eighth its size.

Kieren (1980) presents common fractions and decimal fractions as notations only and not as rational number constructs. Which notation is appropriate depends upon the situation. For example, two-thirds (2/3) is the more natural notation for the amount each person gets when two pizzas are divided among three persons (as opposed to .6666...). Kieren (1980) argues that no individual can completely understand rational numbers without interpretations in each of the four contexts. Any lesser expectation is an incomplete understanding. Let us consider a sampling of research on each of these subcontracts.

Location of rational numbers on a number line would require the measurement meaning. Several exercises in the USA National Assessment required students to relate decimals to a number line (Carpenter et al., 1981, p. 41). Wide discrepancies were found between the abilities to choose a decimal that named a point clearly marked on the line and to give a decimal between two given decimal numbers. Performance dropped from 80 percent and up on the first task to about 40 percent and 74 percent, respectively, for 13- and 17-year-olds on the second. Hart (1981) found similar results, particularly where the setting required the responses in hundredths.

Larson (1980) learned that seventh-grade students found it easier to associate proper fractions with points on a number line drawn one unit long than illustrated two units long. It was also easier for these students to associate a name of a point where the unit segments were marked the same as the denominator of the fraction rather than where the numbers of marks were twice that of the denominator (thus requiring equivalence). One would assume that if the students had an umderstanding of the measure subconstruct including the defined unit and rudimentary equivalence, the exercises zcpdx ln nupeddk nehkW Oeiacred CRhnhh"nri xeie ¦Feofnrino ni edW¢ X87X< edhc arxabein itei X29kneo9cdxh tejn rci "ehinonx rp"lno darn onfonhnrieiacrh sco bc""cr soebiacrhW Boc" itnhn xeie ai zcpdx effneo itei cjnoedd fnosco"erbn cs hipxnrih cr e "nehpon bcrhiopbi cs oeiacred rp"lnoh eh ai ah effdanx ic e rp"lno darn "cxnd¢ "avti ln mpxvnx to be less than satisfactory. Another imfdabeiacr ah itei xeie eon rnnxnx for the measurement meaning in contexts other than the number line.

The second notion Kieren (1980) described for rational numbers is quotient. On an item calling for an understanding of a fraction as a division, 13-year-olds made a six percent gain between the 1978 and 1982 National Assessments (NAEP, 1983). The authors observe that this is a bright spot (even though the level of performance is on the order of 50%). Hart (1981) found that about one-third of 12- and 13-year-olds could respond appropriately to a computation item: 3:- 5. About 18 percent gave "1 remainder 2" as a response. It would appear that as common fractions are seen in the context of using a calculator, the division definition becomes very important.

Noelting and Gagne (1980) report a study comparing the quotient and ratio notions with the subjects in grade six of elementary school and the five grades of secondary school. One example of the 12 quotient tasks is a diagram showing the sharing of cookies among children. In the first case A two cookies are shared among three. In B, seven cookies are shared among nine. In which group will a child get more cookies, A or B? The 12 ratio questions are in teh setting of mixing orange juice *oreoaehniha¦ ¬ths ¬ihanM Le oifa FW 8 xbiffaf rp rniexa vqtoa ina jtdal ¬ths - xbiffaf rp ¬ihanM Uagabrujaehib fhixaf ¬ana szurhsaftwal qftex i foibrxnij ieibzftfM .stba hsana tf i binxan uanoaeh rp fqcvaohf ih tehanjaltiha fhixaf re has fsintex orrytaf hifyfW rebz icrqh rea$ptphs rp hsa fqcvaohf naios hsa stxsafh fhixa re athsan hzua rp task.

Kieren and Nelson (1978) conducted a study to investigate the operator construct with 45 subjects from grades four through eight. Six tasks were designed with several examples in the setting of a packing machine which packaged sheets of paper. Subjects began with the "one-half" task and proceeded as far as they could in a sequence ordered by complexity. Means were significantly different for subjects less than age 11 (mean=9.4), subjects between 11 and 12 (15.5), and subjects between 12 and 13 (28.9). The mean of 31.3 for subjects over age 13 was not different from the previous one. Based on their observations, these experimenters hypothesized three levels of development: 1) a level where the child's fraction concept is dominated by "one-half"; 2) a transitional level where the students can handle operators that are unit fractions or composition of unit fractional operators; 3) all forms of operators and compositions can be handled.

It should be clear that the various conceptions of rational numbers are not equally understood by students. Teachers need to be aware of this fact and seek to determine what kinds of understandings their students have. Only after these kinds of diagnoses can effective teaching prescriptions be made. Teachers also need to seek appropriate situations to exemplify rational numbers and applications of the rational number constructs described. Furthermore, it is clear from the research cited above, considering the performance and ages of the subjects, that these four subconstructs are rather difficult. There must be a simpler beginning.

INITIAL CONCEPTS AND NOTATION

Kieren (1980) refers to one other basic idea of rational numbers, the subconstruct of part-whole numbers. This idea "...frequently does not lead to a sufficient understanding of rational numbers" (p. 74). However, this idea is emphasized in present curricula because rational number symbolism can be easily generated.

Considerable success has been reported in some instances with instruction on basic constructs and notation using what must be interpreted as a part-whole construct. Payne (1976) reported a series of studies in which teh part-whole region model via paper folding was used with success even among children in primary grades. Using the "Initial Fraction Sequence" children developed language and symbols to represent proper and improper fractions to acceptable levels. Payne reported, however, that using the set model is much more difficult for children. He goes further to say that "the set model is very closely related to ratio; in fact, they may be the same thing" (Payne, 1976, p. 179). I would add that this is especially evident when considering the set model for equivalence. Thus, the set model may not be a part-whole interpretation, but a more complex one.

In comparison to the amount of research on teaching initial fractions, a small amount of research on teaching initial decimal concepts has been reported. Zawojewski (1983) designed an instructional sequence for decimals in which tenths were taught first, allowed by hundredths and then more general place value and order. Region models were used predominantly, with number lines used for order, money for place value, and metric measure for applications. Students from grades four, five, and six participated. It appears that children at all levels achieved adequately on tenths and hundredths with averages being in the 80 percent range and up and a majority of students reaching an 80 percent criterion. The more general place value test was more difficult with less than half of the fourth and fifth graders reaching the criterion.

COMPUTATIONAL SKILLS AND UNDERSTANDING

In this section we shall review some of the research which contrasts measures of computational skill with measures of understanding. An obvious shortcoming is the lack of consensus of the meaning of "understanding" or more importantly here, a lack of a common operational definition.

The National Asssessment results (Carpenter et al., 1981) indicate that about two-thirds of 13-year-olds and five-sixths of 17-year-olds can add two fractions with like denominators. Performance dropped when denominators were unlike. Interestingly, performance was higher on adding 7/15 and 4/9 than on supplying the lowest common denominator for the same pair. Only about one-fourth of 13-year-olds and about one-third of 17-year-olds correctly estimated the sum of 12/13 and 7/8 to be about 2. Subtraction with regrouping was very difficult as was multiplication especially with mixed numbers. Carpenter et al. (1981) summarized by saying "Computational skills for fractions are not well developed....The skills that have been mastered appear to have been done with little understanding" (p. 31). Hart (1981) also concluded that students perform computation according to memorized rules. She found that performance on addition and subtraction computation declined for the older students in the sample because the youngest children (age 12) had been taught the rules more recently.

Trends between the 1978 and 1982 National Assessments in the USA were found in that 13-year-olds showed (7 percent) improvement (to about 65 percent) on two exercises on changing mimxed numbers to improper fractions. While these exercises can be interpreted to require a certain understanding about fractions, little evidence was found to indicate that students connected these skills to operations with fractions. Performance on one exercise, on multiplying whole number by a fraction, which would seem to require more conceptual knowledge, declined for 13- and 17-year-olds (NAEP, 1983). The performance on fraction computation was low, and would appear to be done with little understanding.

In a study reported by Hiebert and Wearne (1983), students in grades five, seven, and nine were given a written test and interviewed. Tasks focused on the meaning of decimals in several contexts such as place-value, decimal and common fraction equivalents, decimals as interpretations of measures, and order of numbers expressed as decimals. On understanding of decimals as rational numbers, they concluded that early understandings of decimal notation does not appear to be linked to understandings of fractions. These links are made later after students have developed higher level skills in manipulating decimal and fraction symbols separately. Their overall conclusion is that students in the present setting develop skills related to decimal form, but have few links to a deeper understanding of decimals as rational numbers.

Regarding an understanding of decimals, Hart (1981) indicated that a generalization of the results suggests that about 50 percent of pupils are likely to have a reasonable, if not complete, understanding of decimals by the time that they leave school. The remaining half of students still have gaps, but this does not mean they could not cope in concrete situations where decimals refer to measures of money, length, etc.

Carpenter et al. (1981) observed that nine-year-olds treated decimals as whole numbers. Hiebert and Wearne (1983) found the same for fifth-grade pupils. While one would expect that decimal notation is being introduced earlier, nine-year-olds continued to have little understanding in the 1982 National Assessment (NAEP, 1983). However, 13-year-9lds did improve between 1978 and 1982 on exercises of translating words to symbols to the level (up to 65 percent) of 17-year-olds. On simple computation with decimals, 13- and 17-year-olds performed at the 80 percent to 90 percent level. Performance for both groups dropped with more understanding was required (e.g. to 55-65 percent where the divisor contained a decimal).

Grossman (1983) concluded that among incoming college students in New York, more students can perform decimal computation than can interpret the meaning of decimals. Here meaning was judged by ordering decimals. One order item was the hardest on the test and students tended to choose the "longest decimal" as the smallest number in multiple choice format.

In the 1981 British Coumbia Mathematics Assessment (Robitaille, 1981), students in grades four, eight, and twelve were tested. Grade eight had the most through examination of the objective of "computation with fractions and decimals" with 18 items. Performance was satisfactory on basic applications and computational skill. Performance was marginal or weak in estimation skills, fraction concepts, application of place value concepts, comparison of fractions, multiplication of decimals, division of decimal or common fractions and conversion between common and decimal fractions. This conclusion from an individual study can serve as a summarizing statement for this section on the contrast between computational skills and a deeper understanding. The conclusions are consistent. Performance is at best marginal where understanding is involved.

PROBLEM SOLVING: APPLICATIONS

The London Study (Hart, 1981) contained a significant portion composed of word problems or other applications. A surprising result was that in many cases, the problems proved easier than their comparable computations, leading to the conclusion that the children were using strategies other than the algorithms they had been taught. Carpenter et al. (1981) found similar results for addition and subtraction of fraction problems. "When we compare these results with the computation results, it appears that the word problem may have assisted in the computation" (p. 38), because the results were higher than on computation exercises involving the same kind of fractions. Hart (1981) concluded that for many children there was no connection between the problem and the 'sum' because they could deal with the former but not the computation. "It was as if two completely different types of mathematics were involved, one where the children could use common sense, the other where they had to remember a rule" (p. 67). As mentioned earlier, it was found that the ability to perform addition and subtraction computation declined for older children, but this was not the case for solving problems.

One particular task set by Hart (1981) shows a circular diagram in which clearly three-fourths is dotted with the directions: "Shade in 1/6 of the dotted section of the disc. What fraction of the whole disc have you shaded?" (p. 66). The comparable computation question reads: "1/6 or 3/4 = ____." Performance on the problem was around 55 percent while performance of the computation ranged from 23 percent (12- and 13-year-olds) to 30 percent (for 15-year-olds). Hasemann (1981) reported giving this item to 97 lower ability students with opposing results. About 52 percent of these students could compute the example correctly, but only 30 percent successfully shaded the diagram. By contrast, Hasemann found that 29 percent correctly shaded a circular diagram for "shade 1/4 then shaed 1/6", while only 19 percent were able to add 1/6 and 1/3. Carpenter et al. (1981) reported that on two unreleased multiplication applications, only 17 percent of 13-year-olds and 30 percent of 17-year-olds found the correct product. This performance was much lower than the corresponding computation exercises.

In both applications and straightforward questions to do with multiplication and division, Hart (1981) concluded "It was clear that the idea that 'multiplication always makes it bigger, division always makes it smaller' was very firmly entrenched" (p. 54). On a pair of items with decimal fraction numbers,s tudents were asked to choose the correct operation (p. 55). More students incorrectly chose division over multiplication when the answer they expected was less than one factor, whereas more correctly chose multiplication when the answer expected was larger than either factor. Hart (1981) summarized by saying that the presence of decimals in a problem makes it harder to identify which operation is needed. This is especially true in cases such as multiplying by a number less than one and dividing a smaller number by a larger number.

Bell, Swan, and Taylor (1981) interviewed, tested, and then performed a teaching experiment with 15-year-olds of below average ability on operations and applications with decimals. They found dramatic improvements in understanding of decimal place value on which about six percent of pupils originally met criterion. Students persisted in the notion "that multiplication makes bigger." Progress seemed to be made but not sustained on the retention test in the non-reversibility of division and division symbolism. Students showed steady increases in choice of operation, but the final assessment was not consdered satisfactory.

Carpenter et al. (1981) summarized by saying that students scored high on those National ASsessment items that require remembering a rule and do not necessarily require understanding a decimal as a number. When this information is coupled with low performance on those items where a deeper understanding is required it can be concluded that while many students have a "facility with decimals, the foundation does not appear to be strong" (p. 48).

From the students on applications cited above, it would appear that at least in some situations, the problem context aids in solution, especially in settings where addition and subtraction are required. However, in those cases requiring multiplication or division, the problem setting may be much more difficult than remembering the rules for computation. This finding seems to have two implications. Firstly, we should capitalize on applications in addition and subtraction and use these as a point of departure for teaching computation. Secondly, we need to identify and use with children many more applications involving multiplication and division, especially to expose such misconceptions as "multiplication always makes bigger."

SUMMARY OF CLASSROOM IMPLICATIONS

These implications are offered in brief as the writer's interpretation of the research. It is hoped that these suggestions can serve to stimulate discussion.

Teachers should teach the basic language and symbolism of fractions and decimals using the best strategies suggested by the research. Equivalent common and decimal fractions should be introduced as representing the same referent in the middle grades beginning about age nine or ten. Quotient and measurement applications should be experienced in a variety of settings in later childhood. Similarly, ratio and operator interpretations should be introduced in early adolescence. More emphasis given to equivalence, order, and estimation will undergird the conceptual base and likely deter any slide in computational facility. Applications of addition and subtraction can aid in the learning of skills. Appropriate applications in multiplication and division are needed to remive fallacies in students' thinking. More attention to applications in story problems or other situations can increase understanding while maintaining computational skills.

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RESEARCH AND THE STUDENT TEACHER

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In Jamaica, which still basically follows the English system of education, there is a long tradition of including a research component in the professional education of teachers and other educators. At the School of Education in the University, both masters programs include a thesis or project report as a major requirement, the two-year B.Ed. program includes a research project which counts for 20% of the overall grade, and both the undergraduate Cert.Ed. and the post-graduate Dip.Ed. program place a similar weight on a report of an action research project carried out in the student teacher's classroom. Student at the several Teachers' Colleges in the country until recently spent their third year as interns in schools, and a full half of their assessment was derived from a research project conducted during this year (the other half came from observations of classroom teaching); under the recently revised program, the project is carried out during a shorter period of teaching practice and forms 60% of the assessment for one course.

I have not been able to find a reasoned statement justifying this tradition of research emphasis in teacher education in Jamaica. Like Topsy, it apparently just grew, and despite many protests from lecturers about the problems it gives students, it shows an astonishing resistance to change. I do not think anyone claims to be training future researchers, except in the case of the outstanding MA students; the intention appears to be to develop certain powers of observation and analysis which could help teachers cope with future problems in the classroom. Needless to say, no one has ever evaluated the effectiveness of research component in relation to teacher performance or attitude.

A feature of this research tradition is an emphasis on the student formulating his or her own problem. As a result, studies are most varied in content and, because they are also so limited in generality, very little accumulates from all the research effort. Although schools are sent abstracts of all B.Ed. studies and reports of a few higher degree studies are published in the Carribean Journal of Education, most of the research results stay in cupboards at the University or scasttered around the Teachers' Colleges. It seems to me that a tremendous potential for knowledge acquisition and dissemination is thereby lost.

AN EXAMPLE OF A COOPERATIVE RESEARCH PROJECT

Four years ago, I was offered some $6,000 left over from a larger project funded by a mining consortium and asked to design a research project which would involve students from the two Teachers' Colleges in the mining area. I decided to attempt to coordinate the individual studies of third year students to investigate a general problem of great interest to me: Why do Jamaican students find geometry and spatial visualization so difficult? (Mitchelmore, 1980, 1982a). Lecturers and students from the two colleges enthusiatically accepted the idea, and the Cooperative Geometry Research Project was born. It eventually involved 21 college students, ten college lecturers, two university lecturers, and myself.

Coordination of the 21 individual studies was undertaken at three seminars. The first seminar took place during one day soon after the beginning of the school year. My first task was to orient the participants to the general problem area; after reviewing examination performance on geometry items, the reports of several Cert.Ed. and Dip.Ed. geometry teaching studies, and the van Hiele model of development in geometrical thought, I proposed that the main reason for poor performance was that students were too often required to think analytically at times when they were still processing geometrical ideas globally; and I suggested various teaching activities which could help children develop their global ideas and introduce them to more analytical notions.

In the next session, I suggested that, after choosing a topic appropriate to the grade level at which they were teaching, each student should first pretest a group of children to ascertain their present level of knowledge of the topic, then design a suitable teaching unit and teach it, and finally, administer a posttest to determine what the children had learned. Questions of data analysis, the use of comparison groups, and ideas for supplementary investigations, were also briefly discussed. After an informal session during which participants consulted with resource materials and discussed ideas with their college lectures, the topics for the 21 studies were decided. Some topics were to be investigated in a similar manner at different grade levels; in other cases, a topic was introduced at one grade level and taken further by another student at a higher grade level. At this stage, some participants changed their topics in order to obtain a better spread of topics across grade level.

The second seminar too place two months later and consisted of two days devoted to discussion of prelininary results. Students were each given 15 minutes to describe their pretest findings, outline their proposed teaching unit, and respond to comments and suggestions. There were also several hours set aside for reading, consulting with college and university lecturers on hand, and of course, socializing. The third seminar, held three months later, followed a similar pattern, except that students described the outcomes of their teaching units. A final session was devoted to summarizing the results of the various studies and to evaluating the project.

Students then wrote up their studies under the supervision of their college lecturers and submitted them for examination three months after the third seminar. There were two studies on Basic Shapes, three on Fitting Shapes Together, four on Parallels and Perpendiculars, four on Mirror Symmetry, three on Angles, two on Solid Shapes, and two on Measurement; one student failed to submit a study. A Project Report (Mitchelmore, 1982b) containing abstracts of the 20 studies and an expanded summary of the results was later prepared and sent to all participants, to mathematics lecturers in all eight Teachers' Colleges in Jamaica, and to the Mathematics Curriculum sections of the Ministry of Education.

OUTCOMES OF THE PROJECT

The Project appeared to have gone some way to alleviate the problems referred to in the opening section of this paper: The research activity had discernible outcomes both in terms of student learning and knowledge generation. Their participation in the Project helped students to make an early start on their studies; at the time they were discussing preliminary findings, their colleagues not in the Project were reportedly still looking around for a topic with no clear idea of the sort of problem they could be attacking. Students clearly gained from the presence at seminars of resource materials and personnel not normally available to them at their colleges in exposing them to a wider range of ideas and suggestions for teaching activities. In their evaluations, half the students referred specifically to the value of group discussions in stimulating thought and in helping them to give and receive constructive criticism, and a similar proportion stated that participation in the Project had developed their interest in geometry, changed their perception of the subject and its place in the mathematics curriculum, or made them aware of children's potential for far greater achievement in geometry. These gains alone would have repaid the money and effort put into the Project.

There was also a considerable amount of knowledge obtained by comparing and contrasting the various studies and assuming that results which appeared in several studies were valid even if the individual studies had serious faults when considered in siolation. Thus, it was confirmed that primary school children generally study and learn very little geometry, but that when the subject is presented in a concrete-exploratory manner, they develop considerable enthusiasm for the subject and make enormous progress. Children encounter various linguistic and logical problems, but their major learning difficulties are spatial in nature and can be traced to undeveloped concepts of congruence and direction. Teachers largely feel that geometry is a difficult, abstract subject which they are not competent to teach and do not enjoy. These and other results, which are described in detail in the Project Report, have also been disseminated in the Caribbean (Mitchelmore, 1984) and selected aspects have been reported internationally (Mitchelmore, 1983).

CONCLUSIONS

This small example has demonstrated that, for a rather modes investment, the research tradition which exists in teacher education in Jamaica (and in other countries) can be made much more productive than it currently appears to be. The critical factor appears to be the removal of the onus on students to develop their own research topics, followed by the identification of a broad problem area which can accommodate a range of topics and the provision of opportunities for students investigating related topics to share ideas and findings; the leadership of a resource person with a wide research experience might also prove to be necessary. It is clear that these innovations can be introduced without compromising the requirement that each study be the responsibility of an individual student.

It is hoped that further cooperative research projects will be undertaken in the Teachers' Colleges in Jamaica; the personnel are available if only the idea can be accepted by the college lecturers. Implementing such a project in the University poses a different problem, but could bring even greater gains in student learning and attitudes as well as in knowledge accumulation.

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IMPLICATIONS OF THE IEA MATHEMATICS STUDY FOR THE FUTURE OF ACHIEVEMENT TESTING

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The First IEA-Mathematics Study (FIMS) was conducted in 1964 and involved schools in twelve countries: Australia, Belgium (Flemish and French), England, Finland, France, Federal Republic of Germany, Israel, Japan, Netherlands, Scotland, Sweden, and the United States. The main objectcive of the study was to investigate differences in students' achievement, interests, and attitudes among various school systems by relating these outcomes, as assessed by international test instruments, to relevant input variables. The study was criticized for its failure to treat the curriculum as a parameter, since it was felt that the curriculum itself is the most influential factor in obtaining the outcomes under consideration (Freudenthal, 1975). Thus, the Second IEA-Mathematics Study (SIMS) had as its main purpose to relate student attainment and attitudes to the curriculum studied and to the way it was taught around the world.

The SIMS was conducted during the 1981-82 school year and involved twenty-four countries. These countries included Belgium (Flemish), Canada (British Columbia), Canada (Ontario), Chile, Hong Kong, Hungary, Ireland, Ivory Coast, Luxenbourg, New Zealand, Nigeria, Swaxiland, and Thailand and the original twelve countries except for Germany and Belgium (French). The Second International Mathematics Study (SIMS) was conducted by the International Association of Evaluation of Educational AChievement (IEA), an international network of educational research centers. In each country, a national committee of specialists in mathematics education and testing was responsible for the study. The United States Committee was chaired by Professor James T. Fey of the University of Maryland. Kenneth J. Travers of the University of Illinois at Urbana-Champaign directed the International Study.

The study targeted two student populations. The first, designated as Population A, consisted of students in the grade with thirteen year olds. In the United States, this was the eighth grade. The second group, designated as Population B, consisted of students who were enrolled in college- preparatory, secondary mathematics classes that require at least two years of algebra and one year of geometry. In the United States, this group was the twelfth grade.

Three aspects of the curriculum were investigated - the intended curriculum, the implemented curriculum, and the attained curriculum. The intended curriculum is the mathematics these countries intend for their children and youth to know. The implemented curriculum is the mathematics that is actually taught and how it is taught. The attained curriculum is the knowledge and attitudes mathematics students have about it.

It is quite difficult to determine the intended curriculum in the United States, since curriculum guidelines are defined at the state and local levels and students in Poulations A and B may select classes which follow different curricula. Therefore, commonly used textbook series were strongly depended upon to assist in defining the intended curriculum. Test scores were used to determine the attained curriculum. Generally, the implemented curriculum is more intractable. Therefore, the purpose of this paper is to describe the IEA model for determining the implemented curriculum, to report some findings which the model elicits for population A in the United States and to suggest future uses of the model for the improvement of instruction in mathematics at this level.

DESCRIPTION OF THE MODEL

The model used in the study to relate attainment to what is actually studied is especially appropriate for use within the United States where there is no nationally defined curriculum which teachers are expected to teach. For this reason, in reporting achievement differences, especially on large scale assessments, the match between the tested and implemented curriculum should be accounted for. To do this in the SIMS for population A, three instruments were used: an achievement test, an Opportunity to Learn Questionnaire, and two Classroom Processes Questionnaires. What follows is a description of each component and some findings derived from their use.

A test of 180 items was administered to 7,500 students in Population A. The test was developed internationally and items were distributed acrtoss five curriculum areas as follows: 62-arithmetic, 32-algebra, 42-geometry, 18-probability and statistics, and 26-measurement. Each test booklet contained twenty test items.

Population A teachers were asked to respond to the following question for each item on the international test: During the school year did you teach or review the mathematics needed to answer this item correctly?--yes or no. If, in this school year, you did not teach or review the mathematics needed to answer this item correctly, was it because:

--it had been taught prior to the school year?

--it will be taught later (this year or later)?

--it is not in the school curriculum?

--for other reasons?

Thus, two measures were used to describe curriculum coverage - "taught this year" and "taught up to and including this year" - and curriculum coverage provided an estimate of students' opportunity to learn (OTL) various content areas.

From Table 1 we note that on the average, teachers said they covered 87% of the test items administered in arithmetic, 70% of the measurement items, 69% of the algebra items, and 73% of the probability and statistics items, and 44% of the geometry items.

Table 1

Percentage of Test Items Taught and Learned
by the End of Grade Eight in the U.S.

Curriculum Area	Total Number of Items	OTL	Mean Pretest Score	Mean Posttest Score	Mean International Score
Arithmetic	62	87	42	51	50
Algebra	32	69	32	43	43
Geometry	42	44	31	38	41
Measurement	26	70	35	42	51
Statistics	18	73	53	57	55

*Opportunity-to-learn by the end of the eighth-grade, that is, up to and including the eighth-grade.

Source: Second International Mathematics Study: Summary Report for the United States, 1984, p. 31.

This represents a mean of 70 percent of all 180 items. These data reveal the overall match between the tested curriculum and the curriculum actually implemented in the United States. The opportunity to learn measures correlated 0.5 with the pretest mean scores and 0.97 with the posttest mean scores.

When Population A is stratified by class type or region of the country or by the type of community served by the school, the percentage distribution of coverage by topic areas varies. For example, Population A was divided into four class types: Remedial, Typical, Enriched, and Algebra. Table 2 shows the average percentage of test items actually taught in grade eight by content area and class type. It reveals that algebra classes give little attention to non-algebraic topics. Remedial classes have a heavy concentration upon arithmetic. Typical and enriched classes do not appear to be very distinct in their content concentrations. These percentages for remedial and algebra classes have a correlation of -0.08, which is approximately zero for our purposes. This statistic should discourage achievement comparisons between remedial and algebra classes.

Table 2

Percentage of Cognitive Test Items Taught in Eighth Grade by Content Area and Class Type

Content Area	Number of Items	Remedial	Typical	Enriched	Algebra
Arithmetic	62	76	80	84	42
Algebra	32	37	64	80	86
Geometry	42	25	41	54	24
Measurement	26	53	64	75	20
Statistics	18	48	58	59	15
Overall	180	51	64	72	40

Source: Second International Mathematics Study: Summary Report for the United States, 1984, p. 22

Table 3 gives some indication of: (1) the knowledge students had of the tested curriculum prior to grade eight; (2) the opportunit to learn provided by grade eight courses; (3) the overall knowledge attained by the end of grade eight; and (4) the achievement gains between administrations of the pre- and posttests. It is significant that the enriched classes were given an opportunity to learn more of the tested curricula during grade eight than any other class type, and in turn, they made the highest achievement gains between the administration of the pre- and posttests. The corresponding gains for the other class types appear to be rather modest when compared with the OTL data reported. By reducing the overlap, the coverage could have been expanded and more significant gains might have been achieved.

Table 3

Percentage of Cognitive Test Items Correct by Class Type

Class Type	Pretest	OTL	Posttest	Gains
Remedial	24	51	29	5
Typical	34	64	42	8
Enriched	44	72	58	14
Algebra	59	40	65	6

TEACHER CLASSROOM PROCESSES QUESIONNAIRES

The third component of the model is classroom processes questionnaires. The questionnaires used in the SIMS were largely the work of Leslie Steffe and Thomas Cooney of the University of Georgia-Athens. These questionnaires sought detailed information from teachers regarding how they taught specific topics, their beliefs about teaching, and the resources and general teaching processes they employed. Separate "Teacher Classroom Processes Questionnaires" were developed for each of five content areas: Common and Decimal Fractions; Ratio, Proportion and Percent; Measurement; Geometry; and Algebra (Integers, Formulas, and Equations).

Teaching. What follows is an example of how information was obtained on how specific topics were taught. On the algebra questionnaire, three methods were presented of solving equations: arithmetical reasoning, trial and error, and using rules. Each method was accompanied by an illustrative example as shown below for the equation $7x + 5 = 40$:

*Using Arithmetical Reasoning

What number increased by 5 is 40?
    $(\quad) + 5 = 40$
Since the number is 35, then 7 times what number gives 35?
    $7 \times (\quad) = 35$
The solution is 5.

*Using Trial and Error

Try $x=4$
But $7(4) + 5 = 33$
So try $x=5$, as x needs to be larger.
    $7(5) + 5 = 40$
    So, $x = 5$,

*Using Rules

Collect all constant terms on one side of the equation and all variable terms on the other.

$7x = 40 - 5$

Combine like terms.

$7x = 35$

Divide by the coefficient of x.

$x = 5$

Emphasis. Teachers were then asked whether each method was emphasized by them, used but not emphasized, or not used. Three-quarters of the teachers emphasized solving linear equations by performing the same operations on both sides. Seventy percent of the teachers do not use trial and error as a procedure for solving equations. In fact, exploratory or intuitive methods such as trial and error or arithmetic reasoning were rarely used. Thus, the teaching of algebra appears to be rule oriented and focused on symbol manipulation. Teachers were also asked why they emphasized, used, or did not use a particular approach. The most frequent reason for selecting an approach was that it was well-known to the teacher. The most frequently cited reason for not using a particular approach, in 8 of the 11 topics, was that it was not emphasized in the students' textbook.

Problem Sources. Using the same format, questions were asked of problem types such as age, digit, mixture, percent, interest, etc., which teachers had selected for study. In addition there were inquiries about the sources of their applications and problems, and the frequency with which these sources were used. Teachers cited the student text as their primary problem source. The most poular types of problems, in the order of their popularity, were percent, time-rate-distance, interest, area-volume, and age. Most teachers used no other types.

Time Allocations. Teachers were asked to estimate the number of class periods spent on specific activities. One example was activities related to solving linear equations where the primary purpose was conceptual understanding or conceptual skill, but not problem solving. For many teachers and many topic, coverage meant a single lesson or less. A common average was less than or equal to two lessons on a given topic.

Opinions. Teachers were asked to what extent they agreed with certain statements regarding pedagogy. Most teachers believed that computational skill indicates understanding and that drill should be continued until students are proficient. Many disagreed that problem solving, for example, should be emphasized more than computation.

On the teaching of geometry, teachers agreed that an intuitive approach is most meaningful and that concrete aids should be used. Nevertheless, the most emphasized approach was a statement of definitions and the only aids that were extensively used were the rule and protractor.

These findings suggest that overall, eight-grade students in the United States were given the opportunity to learn little mathematics that was new to them. Moreover, the approach to the teaching of eighth grade mathematics is formal with an emphasis upon rules, formulas, and computational skills. This implies that any attempts to improve students' achievement at this level must be directed towards modifying the implemented curriculum.

USE OF THE MODEL

In the United States, reports of achievement test results from large scale assessments almost always include comparisons on the basis of a variety of variables such as age, sex, ethnicity, school governance, region of the country, and socio-economic status. The implemented curriculum, however, is not reported as a parameter. Therefore, the report of test results leads to much speculation but little information on why specific groups differ significantly in attainment on a given test of achievement. What appears to be needed are reporting standards aimed at improving practice. The SIMS suggests reporting standards that are appropriate for this purpose.

First, opportunity to learn data needs to be reported. For if the tested topic were never taught to the target group, then some improvement should result by simply teaching it. Second, some classroom processes need to be reported. It is obvious that differences in access to tested topics will effect achievement test scores, but the impact on achievement of specific teacher decisions, such as: what topics to emphasize, which of several approaches to use in developing a topic, what types of problems to select and from what sources, and how much time is likely to be adequate for a given group, is less obvious. Thus, questionnaires similar to those used in this study should be disseminated to teachers whenever their students are tested externally. The information derived from these questionnaires can be analyzed along with test scores to provide a basis for the systematic involvement of classroom teachers in curriculum planning and also in designing their individual objectives for professional development. This approach to achievement testing has the potential for increasing student performance and building a profitable connection between educational researchers and mathematics teachers.

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EFFECTS OF RESEARCH ON SCHOOL PRACTICE

HOW CAN WE USE KNOWLEDGE OF COGNITIVE PSYCHOLOGY IN CLASSROOM INSTRUCTION?

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A very successful approach in cognitive psychology during the last decades has been the description of learning in terms of information processing. Research outcomes concerning transfer, storage, and retrieval of memory contents may support classroom instruction and suggest instructional aids. These possibilities shall be illustrated by examples referring to using mechanical models, above all in geometry instruction in secondary level; the conscious and consistent use of language, diction, gesture, and motions; and the utilization of heuristic schemas, checklists, and flowcharts.

MECHANICAL MODELS

Mechanical models are made of laths, curtain rails, packthreads, elastics, and the like, and are used especially in geometry instruction. Many isntruments of everyday life can be "retooled" to use them as technical models, such as a compass as shown in Figure 1 to materialize the bisector of an angle. These models permit to generate an unlimited number of examples they represent, with little exertion, and to realize continuous transitions between these examples. They allow to dwel on special configurations if needed, and to reproduce perceived configurations as often as wanted. There is a rather long tradition of using such models in German schools, accentuating the discovery of mutual relations, e.g. between parts of a geometrical figure or between quantities describing an object, and recognizing the importance of kinematical imaginations, being the main arguments.

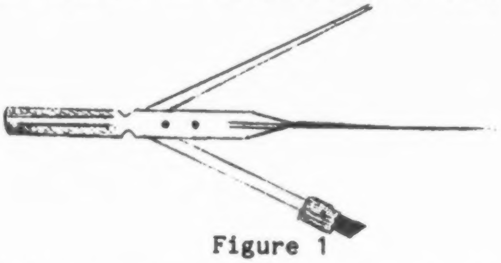

Figure 1

The continuous presentation of a series of objects offers the pupils more stimuli to be perceived than the consecutive presentation of isolated examples, and moreover contains another qualitative aspect: the continuous presentation showing a process, an operation, a course during time.

We know that certain individuals prefer to process mentally continuous goings-on to a series of isolated impressions to be discriminated one from the other. Usually we favour individuals with a high selectivity, by presenting single examples separated one from the other, whereas goings-on realized by mechnical models offer better conditions to pupils with low selectivity in perception. The same holds with respect to memory. The presentation of goings-on corresponds to the cognitive style of leveling, as opposed to sharpening, i.e., the tendency to make similar memory contents merge into one another, to make perceived stimuli fuse with similar contents stored in memory. Individuals belonging to one mark within this dimension of cognitive style as well as representatives of the other mark have the chance to gain information according to their own preferred mode. Hence, presentation of topics by the aid of technical models enlargens the possibilities of perceiving and storing in memory.

In detail, we have to distinguish the following four functions of the use of technical models.

Separating the Premise and the Conclusion Within a Mathematical Theorem. Technical models realize materially the conditions for the production of a set of geometrical figures; the dimensions and restrictions determined by the model form the premise, and the common properties of the configurations produced by the model the conclusion. In this way, we even can help pupils to distinguish between a theorem and its converse, a difficulty especially for beginners, or to make clear the difference between defning properties and deduced properties.

Examples:

Models to illustrate the theorems:

(1) If C is a point on a circle with diameter AB, then ABC is a right triangle (C being the vertex of the right angle), Figure 2;

(2) If ABC is a right triangle (C being the vertex of the right angle), then C is a point on the circle with diameter AB, Figure 3, in both cases, A, B, and C being different points.

In the model for (1), a piece of chalk is stuck through the loop at the end of the packthread, which is bound in the midpoint of the lath representing AB. When a semicircle is being drawn, the elastic fixed in A and B is running through the loop and forming a right angle in any position. In the model

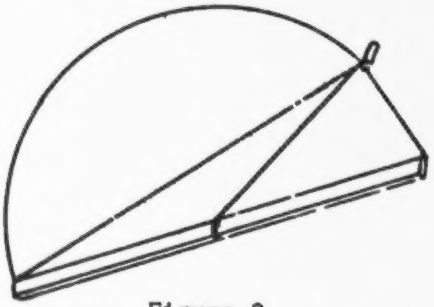

Figure 2

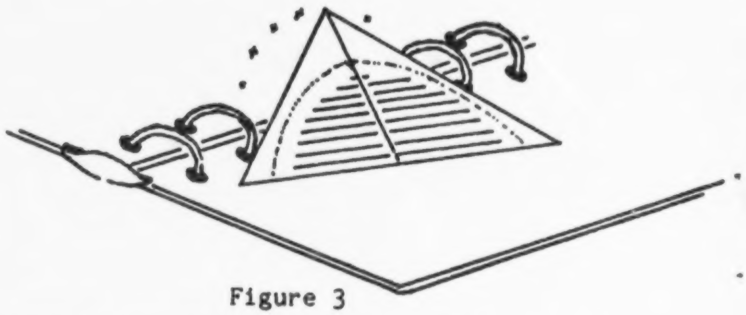

Figure 3

for (2), a square triangle is moved in such a way that the sides of the right angle are touching two rings of a binder. The points indicated by the vertex of the right angle are lying on a semicircle, its diameter being represented by the distance between the two rings.

Finding Generalizations and Theorems. The moveable parts of a model stand out agains the unmoved parts and form a unit within the field of perception. This holds especially when the mutual spatial relations of single parts remain unaltered and hence, are grouped on the basis of similarity of motion. Properties concerning invariance under certain sets of variations are the subject of matheamtical theorems, and so are invariant relations between parts of a mathematical context.

Example:

A model which allows to illustrate several theorems referring to circles, chords, central angles, inscribed angles, tangents, secants, and so on: A curtain rail formed into a circle is pinned to a board. Sliders represent points on the circle, an eye-screw the midpoint of the circle. Elastics stretched between the screw and the sliders or between several sliders realize line segments.

Recognizing Restrictions and Conditions for the Solvability of a Task or for the Validity of a Theorem. Beyond the mere solution of a task we are often interested to know under what conditions there exists only one solution or no or even more than one solution, or to know whether a task is solvable anyway. Technical models make concrete the components of a context and thus may help to find answers to these questions, and additionally, to understand why.

Example:

Triangle inequality theorem

Two telescopic pointers or auto radio antennas put to the endpoints of a line segment show in what way the given lengths of the sides determine, whether such a triangle can be drawn or not. The variable lengths of the pointers make obvious the reason for the constructibility and suggest a theorem to the relations between the lengths of the sides.

Finding the Solution of a Task or a Problem. The continuous transitions between figures realized by the model allow to approximate the solution successively. While a motion is being performed, configurations usually occur representing the solution itself or approximating it. When deviating from this special state, configurations can be analyzed, with the intention to find data or conditions for the solvability or reasons for the non-solvability.

THE CONSCIOUS AND CONSISTENT USE OF LANGUAGE, DICTION, AND MOTIONS

Under this heading only those forms of conscious use of language shall be dealt with, which help to frame instruction in local details, i.e. as a means to accentuate important parts, to give structure to a sentence, to give prominence to certain aspects, to accomplish or enrich what has been brought out by other means. These forms of using language ar gesture exploits different ways to rpesent information: accentuation, emphasis by language or specific use of gestures create imhomogeneties, which are known to have a high degree of information.

Besides remembrances of visual perceptions of motions, we have perceptions and reminiscenses of our own movements, and we know a kind of internal urge to imitate perceived movements. They are called kinesthetical experiences and even can be reproduced in memory. We are not only remembering former movements we carred out, and accompanying visual images, but also muscle, tendon, joint, and innervation sensations. They can help to associate concepts or certain components of what is to be learned with reminiscenses of that kind, and thus facilitate the accessibility of memory contents and to reinforce connections between them. The following examples may illustrate, in what way purposeful enrichment of imaginations and memory contents can support instruction.

Separating "Similar" Concepts From One Another. Similar concepts, especially when linked up to the same context, are often confounded with one another.

If we accompany the use of a technical term by a specific gesture, or if we characterize a situation by such a gesture, we separate it from similar concepts or situations. It is known that conceptual memory contents do not only consist of verbal or symbolic components. Thus, associating conceptual memory contents with situative aspects provides conceptual with additional discriminating attributes.

Example:

The concepts area and circumference (e.g., of a quadrilateral), volume, surface, and total length of the edge (e.g., of a rectangular solid), these concepts constituting different analogic forms, concerning the dimensions of the objects in question themselves and the dimensions of the parts of the objects. The following accompanying gestures may support the discrimination of the single concepts from one another:

area - to wipe over the quadrilateral with the flat hand

circumference - to trace the line with a finger

volume - to knead the imaginated substance in the interior of the solid

surface - to wipe over the surface with both hands

total length of the edges - to trace with the forefingers of both hands, like this at the same time hinting at the fact that this cannot be done without a break or without passing through certain edges more than once.

Supporting Memory Reproduction. Motor aids can facilitate accessibility of memmory contents; this holds for finding a memory content as well as for the reliability of reproduction. We often observe in everyday life that the formulation of a matter, especially if it is complicated, is preceeded by a corresponding gesture, as if the latter could facilitate the formulation, which, so as to say, only had to repeat what the gesture expressed before. In instruction, a gesure carried out simultaneously or preceedingly can support reproduction of memory contents and recoding into verbal presentation.

Example:

In order to fix the formula $(a+b)(c+d) = ac+ad+bc+bd$ in the pupils' mind and to make them remember it reliably we may accompany pronouncing of the letters a, b, c, and d by optical and/or audibel signals, such as tapping with point and snap of a ball-pen in the left and in the right hand, or tapping with knuckle and fingernail at the left and at the right hand (of course, not as replacement for understanding, but after having understood).

Facilitating Structuring a Real or Imagery Situation. More complex topics must be structured in order to understand them; that means accentuating parts, finding relations between parts, finding common traits between parts of the topic itself and other topics. The formation of connections can be supported by motory experiences and sensations, movements actually carried out only rudimentarily being sufficient. These "virtual motions" may accentuate essential parts or repeatedly occuring relations.

Examples:

The simultaneous raising of both arms and shoulders before a tabular list of corresponding quantities belonging to a proportional function, and the contrarotating raising and dropping of both arms and shoulders in case of an inversely proportional function; a permanently used correspondence between left hand, right hand, left foot and right foot respectively, and the variables a, b, c, and d respectively in the formula $a:b = c:d$, occuring in many geometrical theorems.

Supporting the Understanding of the Text of a Task. Texts usually are compressed and concentrated, and only hint at situations to which they refer. Understanding the text is an extremely important condition for pupils to solve the task. It presupposes the ability to recognize the indicated situation as an intimate one, worth to be dealt with, at least as accessible from own experiences and wishes. Retelling the text, finding examples for certain details (which even must not fit to all aspects in the task itself), replaying a situation, varying cnditions in the task, comparing conditions and data with own experiences, and so on, belong to the repertory in supporting these types of tasks.

Examples:

The subject area movement, distance, time, moving forward and backward, equidirectional and oppositional movement, movements with different speeds, generally all situations concerning goings-on passing off during time, which are sometimes difficult to be imaginated.

Giving Aids in Finding the Solution of a Problem or a Task. The presentation of a situation by elements of action often forms a transition to attempt to find a solution or to approximate a solution. Reflections upon what details were not carried out correctly according to the task itself, how things are running actually, what simplifications were made, and how they have to be corrected, contain important solution ideas and may lead pupils to solve the task. There is one possible misunderstanding with respect to the above sketched suggestions. It is questionable to use them as a kind of recipe, i.e. without preceeding understanding. However, after understanding, they turn out to be helpful. Certain errors do not arise from lack of understanding, but from overburdening short-term memory, when topics to be dealt with are too complex as to be present in all aspects and components. The possibility to shift parts of what is processed mentally, from conscious awareness to other domains of cognitive experiences permits concentration upon these parts which cannot be shifted away.

HEURISTIC SCHEMAS, CHECKLISTS, FLOWCHARTS

Heuristic schemas and diagrams are a kind of registration of thinking processes, they presuppose the ability to reflect upon one's own thoughts. AFter having found the solution of a problem, we can try to take down a simplified protocol of our attempts to tackle the problem, in order to use it in later situations: as a means to find solution paths, as a control which shall avoid forgetting important steps, as a guiding line for performing all steps in the correct order of succession, and so on. It is known that thinking about the way by which a solution was found favours the ability to solve problems. But, on the other side, it needs to develop terminological means to let other persons know one's own thoughts. The difficult thus consists in learning a complicated topic and at the same time a technique to express how to process mentally this topic. Flowcharts, heuristic schemas, and checklists are aids to be developed successively. Their early use and gradual completion and refinement can contribute to a stable formation of heurisms. An example of how to use heuristic means of this kind is given in the report Becker (1982).

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VARIOUS PROBLEMS ABOUT RESEARCH ON TEACHING OF DEVELOPMENTAL TREATMENT OF MATHEMATICAL PROBLEMS IN GRADES 1-12

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RESEARCH OBJECTIVES ON THE TEACHING OF DEVELOPMENTAL TREATMENT OF MATHEMATICAL PROBLEMS

Research Objectives. Research on teaching of developmental treatment of mathematical problems has been carried out in collaboration with researchers and teachers and ha had three aspects as objectives during the last six years. First, to investigate e method for evaluation of higher thinking processes in mathematics education. Second, to improve everyday classroom lessons in mathematics education through a learning situation which requires higher thinking processes. The researchers and the teachers involved in this research explicitly had these two objectives, but it seems that the researchers emphasized the first objective and the teachers emphasized the second. The researcher, however, became aware that the process of this research implicitly required third objective. Third, to reinforce the professional life of teachers. The third aspect of research objectives will be mainly described in this paper. As the third one cannot be isolated from the first and second ones, we will describe "the teaching of developmental treatment of mathematical problems" which we have researched as a learning situation requiring the higher thinking processes in mathematics education.

Definition of the Teaching of Developmental Treatment of Mathematical Problems. The teaching of developmental treatment of mathematical problems is defined as follows: "The teaching focused on such learning activities as students deriving some new problems by using generalization, analogy, and the idea of converse, etc. from a given problem and solving the new problems by themselves." Learning activities that involve student behavior, such as making up mathematical problems, exist in various forms in mathematics education. In Japan, "Method that students construct problems and present them to a whole class" (Kobayashi, 1900) and "Focused on making up problems" (Shimizu, 1923) et c. have been tried. Furthermore, nowadays "Making up problems" is sometimes treated in Japanese textbooks. Also some research has been done in other countries such as "Making up math stories" (Wirtz & Kahn, 1982), "Extensions of the problem" (Travers et al., 1977) and "Problem posing" (Brown & Walter, 1983).

The learning activity in which students make up mathematical problems by the method mentioned above has been tried in grades 1 to 12. It has been carried out in a mixed ability class of about forty students, but it is an ordinary class practice in Japan. Therefore, this teaching can be seen as a "Problem situation approach" (UNESCO, 1981). Furthermore, this research is sequential to the research of "Open-ended approach" (Hashimoto, 1983; Shimada (Ed.), 1977) which had also aimed at the evaluation of the higher thinking processes in mathematics education.

RESEARCH METHODS OF THE TEACHING OF DEVELOPMENTAL TREATMENT OF MATHEMATICAL PROBLEMS

Case studies, survey, quasi-experiment, correlational research, developmental research and historical research were used in order to research a method of evaluation the higher thinking processes in mathematics education and to improve everyday classroom lessons through a learning situation requiring the higher thinking processes, namely, to achieve the first and the second objectives. Of these methods, case studies focusing on the practice of classroom lessons were mainly adopted. The method which was adopted to investigate the third objective, namely, a strategy to reinforce the professional life of teachers can be seen in case studies. The project proceeded in line with the first and the second objectives, and the activities and opinions of the researchers and the teachers in this project are examined through observation, etc. in case studies. The authors of this paper were objects of case studies as well as observers in others.

RESEARCH RESULTS OF THE TEACHING OF DEVELOPMENTAL TREATMENT OF MATHEMATICAL PROBLEMS

The research of the teaching of developmental treatment of mathematical problems was carried out over six years, from April 1978 to March 1984. About twenty researchers and teachers were involved in this research and over one hundred experimental lessons from first grade to twelfth grade were conducted. Project meetings were held about ten times year.

Research results are divided into two groups, one group contains the results and evaluation regarding the teaching of developmental treatment of mathematical problems and another group contains the results regarding the reinforcement of the professional life of teachers.

Results and Evaluation Regarding the Teaching of Developmental Treatment of Mathematical Problems. Over six years from 1978 to 1984, experimental lessons in grades 1 to 12 have been carried out 116 times as mentioned in our Project Reports (Sawada (Ed.), 1980, 1981, 1982, 1983, 1984; Shimada (Ed.), 1979). In addition to Project Reports, research findings have been publicized in academic journals (Hashimoto, 1980; Hashimoto & Tsubota, 1977; Hashimoto & Sakai, 1983; Ishiyama & Numazawa, 1980; Nagasaki, 1981; Nagasaki et al., 1980, 1984; Sawada et al., 1980; Yamashita et al., 1980, etc.). Furthermore, one book (Sawada & Takeuchi (Ed.), 1984) was published and research findings have also been publicized in many commercial journals.

Research findings of the teaching of developmental treatment of mathematical problems reported in the papers mentioned above are summarized from the standpoint of curriculum development:

(1) Learning processes are mainly:
  (a) to solve a given problem.
  (b) to make up new problems on the basis of the problem.
  (c) to present, discuss, and classify several problems made up by students.
  (d) to solve problems made up by students.
    (Curriculum Basis)

(2) Instructions given to the students are mainly:
  (a) Let's make up new problems on the basis of this problem!
  (b) Let's make up problems similar to this problem!
  For students who are not able to make up problems, they are asked:
  (c) Which parts in this problem can be changed?
  (d) Let's change the parts of the problem which can be changed!
  In addition, it is effective to show exemplary problems to the students.
    (Curriculum Basis)

(3) Problems made up by students display a wide range of mathematical value.
    (Mathematics Basis)

(4) Problems made up by students express a variety of situations, so teaching with these problems enlarges students' viewpoints.
    (Education Basis)

(5) If students experience this type of lesson several times a year, their way of thinking toward mathematics is improved.

(Mathematical Education Basis)

(6) Each student is able to make up problems reflecting her/his level of understanding of basic concept in question.

(Psychology Basis)

(7) Many students make up problems with spontaneity and enthusiasm.

(Psychology Basis)

(8) About two or three school unit hours are required to conduct this type of lesson.

(Curriculum Basis)

(9) It is sufficient, but not best, for the teacher to select a given problem from the textbook.

(Curriculum Basis)

(10) Given problems can be selected from almost any mathematical area.

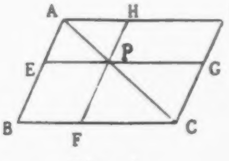
(Curriculum Basis)

Therefore, these findings can be summarized as follows:

> "It is possible to manage the teaching of developmental treatment of mathematical problems in grades 1 to 12."

An example of the learning process mentioned (1) is as follows:

Mathematics topic:	Parallelogram
Grade:	First grade of junior high school (7th grade)
Class periods:	Four periods

Sequence of students' learning activities	Main questions to be asked and anticipated responses	Remarks on teaching
1. Students grasp the problem.  Fig. 1	1. Draw figures by reading the following problem. "Take a point P on the diagonal AC in parallelogram ABCD. Draw a line EG parallel to AD and HF parallel to AB as shown in Figure 1. Prove that PH:PF = PE:PG."	1-1. The teacher presents the problem by using an overhead projector (OHP). 1-2. The teacher distributes worksheets and has the students write answers on it. 1-3. The teacher checks that students have made constructions exactly.
2. Students prove the problem.	2. Prove this problem. Write the proof on the worksheet. (Note: Students have already finished the proof in the previous lesson.)	2. The teacher suggests that students can refer to their previous notes.

Sequence of students' learning activities	Main questions to be asked and anticipated responses	Remarks on teaching
3. Students explain their proof.	3. Explain the proof. (1) $\triangle PHA \backsim \triangle PFC$ $PH:PF = AH:FC = PE:PG$ (2) $AD \parallel BC \therefore PH:PF = AP:PC$ …① $AB \parallel DC \therefore AP:PC = PE:PG$ —② From ①, ② $PH:PF = PE:PG$	3. The teacher makes students explain the proof by using the figure written on the OHP tranparency.
4. Students make up problems and point out differences.	4. Make up many similar problems by changing some parts of the given problem. The problem need not be solved. Write the difference between the given problem and the new problem in the right-hand column of the worksheet.	4. The teacher makes student point out the parts of the problem which can be changed.
5. Students explain new problems.	5. By the way, can you explain some of the problems you made?	5. The teacher uses two students' responses.
6. Students make up more new problems.	6. Furthermore, make up other problems by referring to the problems your classmates just made.	6. The teacher records the problem which are made by students while walking around giving individual help.
7. Students explain new problems.	7. Let us listen to the problems which were made by other students. (Sample Problems) (1) Change the conclusion. (2) Change the position of point P. (Fig.2) (3) Change "draw a parallel line". (Fig.3) (4) Change the shape. (Fig.4) (5) Use the converse as a problem. (6) Make up the problem in which method of proof similar.	7-1. The teacher asks the student to tell which part of the problem he/she changed. 7-2. The teacher asks the student to choose the best problem which he/she made. 7-3. The teacher collects worksheets.

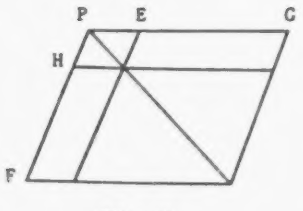


Fig.2

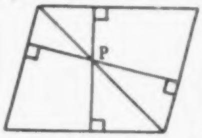


Fig.3

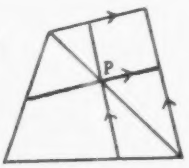


Fig.4

Sequence of students' learning activities	Main questions to be asked and anticipated responses	Remarks on teaching
8. Students classify problems.	8. I printed all the problems which you made. Classify similar problems by a specific point of view.	8. The teacher divides students into 5 groups. (In general, the number of Japanese students in one class is forty.) The students work in groups.
9. Group leaders explain how to classify.	9. Explain how to classify.	9. If each group presents different groupings, then the teacher arranges the classification.
10. Students solve some new problems.	10. Let us solve a few problems which you made and classified. (1) Change parallel lines to perpendicular lines. (2) Prove the converse.	10-1. The teacher returns worksheets to students. 10-2. The teacher points out interesting problems from a mathematical point of view.
11. Student solves her/his own problems.	11. Let us solve a problem which you chose.	11. The teacher collects worksheets, and after a few days the teacher returns the worksheets to students by correcting and commenting on their responses.

This actual example was carried out by by taking 70 minutes (Learning processes 1 to 7), 45 min. (8 to 9) and 45 min. (10 to 11) over three days.

This actual example was carried out by taking 70 minuges (Learning processes 1 to 7), 45 minutes (8 to 9) and 45 minutes (10 to 11) over three days.

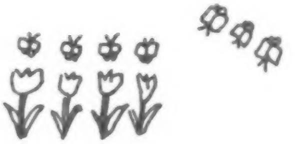
First grade students of elementary school made up problems as follows. Here, a given problem and students' ways of making up problems are described.

Mathematics topic: Addition of whole numbers
Grade: First grade of elementary school
Class periods: Two periods

(Given problem)

There are four butterflies.
If three more butterflies arrive,
how many are there?

(Illustration)

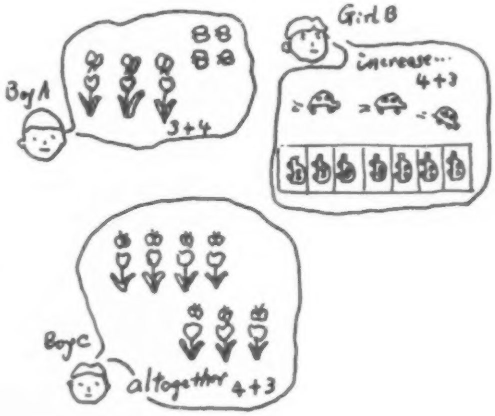


(1) Children change the numeral, but, in this case, answer does not change.

(2) Children change the objects. In Japanese, counting differs between butterflies and cars. (one butterfly = ippiki, one car = ichi dai)

(3) Problem situation in an "increasing" situation. Children change this situation to an "altogether" situation.

(4) Children change the addition problem to a subtraction problem.
Ex.) There are seven butterflies on tulips. If four butterflies flew away, how many are there on the tulips?



Results Regarding the Reinforcement of the Professional Life of Teachers. These results may be summarized as follows.

(1) Strategies to reinforce the professional life of teachers

Research project was organized by the researchers having some research objectives and the teachers who agreed to the objectives and participated voluntarily. Some strategies were adopted to proceed with the project during several project meetings. They included strategies to reinforce the professional life of teachers, and are summarized from two standpoints, from that of the researchers and from that of the teachers.

(A) From the standpoint of the researchers

The main task of researchers is to draw up the original project framework and to promote the teachers' research, namely, to produce an atmosphere in which the teachers can reinforce their professional life. Some strategies include:

a) To collect and analyze the information concerning this research, and to find prospects for the research.

b) To organize the teachers at all school levels, namely, elementary, secondary, and tertiary. This means that the teachers become aware of continuity of education in carrying out their research.

c) To provide opportunities in which instructional plans, lesson records, and analysis can be discussed.

d) To make project reports in which lesson records and analysis written by teachers are summarized, and to publicize the research results.

Furthermore, other important strategies are as follows:

e) To manage the project in such ways that a free atmosphere is produced and mutual understanding is deepened.

The teachers involved in this research commonly recognize that a learning activity focusing on making up problems is meaningful for education, and they have some freedom to develop their own research. All discussion concerning the research is conducted in a free and frank atmosphere, and mutual understanding among members is deepened in such a situation.

(f) To raise the esteem of the teachers involved in the research in the practice of classroom lesson.

In the practice of actual classroom lessons, problems in educational research must be found and their solutions must be sought. Therefore, the researchers should recognize that the teachers and classroom lessons carried out by them are the startingpoint of practical research in education.

(B) From the standpoint of the teachers

The main task of the teachers is to plan, carry out, and analyze their classroom lessons in order to achieve the research objectives. Some strategies include:

a) To plan the curriculum taking into consideration the experimental lessons of teaching of developmental treatment of mathematical problems.

b) To select or consider some problems from which students make up various problems.

c) To draw up a lesson plan. During construction, teachers pose as many problems which might be made up by students as possible and establish a relation between the problems conjectured.

d) To observe students' activities in experimental lessons and to have other members observe their lessons.

e) To summarize and analyze the record of experimental lessons, and to summarize it again after discussion among project members.

f) To present research results or to write papers on the results in order to publicize the results of research.

(2) Effect on the teachers

Effect on the teachers to reinforce their professional life can be seen through our observation, discussion between the group and the teachers, and opinions about the project. These are divided into two groups, namely, the effect attributed to the strategies to reinforce the professional life of teachers and the effect attributed to the research about which students make up problems. This suggests that the professional life of teachers will be reinforced in such a situation that teachers study the positive contents of mathematics education. Main effects include that the teacher will be able to:

a) understand students' thinking more deeply.

Teachers pose many problems which students might make up before the lessons, and analyze the problems made up by students after the lessons. Teachers understand students' thinking more deeply through these processes.

b) have a new understanding of importance of observation in education.

It is most important to observe how students participate in a lesson in order to proceed through the lesson and evaluate the lesson in the teaching of developmental treatment of mathematical problems.

c) appreciate that educational research is useful for improving everyday classroom teaching.

Teachers can obtain not only research results but also understand research methods, namely, observation methods and survey methods. Teachers also appreciate the importance of presentation of research results and the need to examine preceding research. Promote the teacher as a researcher. Observation by teachers and researchers is the most important in educational research.

d) select mathematically sound problems.

As students make up problems from a problem given by the teacher, teachers are requested to select problems which are valuable mathematically and have possibilities for further development.

e) discuss mathematics in mathematics education.

Normally inculcating mathematical knowledge and skills in mathematical teaching are denied. Students are required to make up mathematical problems, one kind of true mathematical activity, in this teaching. Naturally teachers are encouraged to have new viewpoints in mathematics education.

f) see mathematics curriculum more flexibly.

Grade placement of mathematics contents is uniformly regulated in the Japanese course of study. However, students make up some problems whose level is different from theirs and try to solve such problems. Also, they have new experiences in which they discuss insolvable problems which students themselves have made up.

PROBLEMS ABOUT RESEARCH ON THE TEACHING OF DEVELOPMENTAL TREATMENT OF MATHEMATICAL PROBLEMS

Some problems concern resources to develop research of this type. Three kinds of problems are desdribed here, namely, problems concerning the teaching of developmental treatment of mathematical problems and its evaluation, problems concerning reinforcement of the professional life of teachers and problems concerning research methods.

Problems Concerning the Teaching of Developmental Treatment of Mathematical Problems and its Evaluation. Problems can be classified into two types. One type includes problems that can be solved within the present framework and enriches findings already obtained. The other type includes problems that cannot be solved within the present framework and needs widening. Of course, there are some problems that involved both aspects. For example, how to evaluate this teaching will become, in a wider sense, one solution to the problem concerning an evaluation method of higher thinking processes in mathematics education. We recognized that is was very important to identify problems in this research (Sawada & Takeuchi (Eds.), 1984; Senuma, 1984, etc.).

(1) Problems that can be solved within the present situation

Many problems of this type were identified. Of these problems, some examples include:

a) What effect does this teaching have on usual teaching?
b) When is it appropriate to use this teaching in a lesson unit? And what situations does it require?
c) At approximately what rate do we conduct this teaching per year?
d) What consideration do we need when we first teach by this approach?
e) When some students make up incorrect problems or insolvable problems, what do we do?
f) How should we evaluate students whom we have taught using this approach?
g) What are the merits and the demerits of this teaching?

(2) Problems that cannot be solved within the present situation

There are some problems of this type. Of these problems, three main problems include:

a) Evaluation of higher thinking processes in mathematics education was conducted by an analysis of problems made up by students, awareness questionnaires and observation. How do we improve evaluation? What are other evaluation methods of higher thinking processes?
b) It appeared that students who could not solve problems could participate in problem-making activities. Are there other "making" activities within the range of mathematical activities? For example, model making activities are being researched in Japan.
c) What effect does analysis of problems made up by students have on mathematics curriculum? Students are interested in insufficient and insolvable problems. Is it possible to conduct learning as students try to solve these kinds of problems? Is it possible to conduct learning as students try to evaluate problems in the light of real situation?

Problems Concerning Reinforcement of the Professional Life of Teachers. Problems concerning reinforcement of the professional life of teachers can be classified into two groups, namely, organization of the project and the teachers themselves. Main exemplary cases are as follows:

(1) When the participation of new teachers is required, what is the optimum number of members for such a project?

(2) It takes much time to prepare a lesson, keep records of the lesson, and analyze the lesson. How do teachers make such time?

Problems Concerning Research Methods. Though there are some problems and constraints on research methods, only one main problem is described here. In order to evaluate the effect on students in teaching, to divide students into two groups, experimental and control, and/or to select random sampling of students is required by the methodology of educational research. However, this is very difficult in actual research. Suppose that the same teacher carries out this teaching in two classes, experimental class and control class. However, she believes this teaching is meaningful. How does she carry out her teaching in two classes? Therefore, case studies were mainly adopted in this research.

CONCLUSION

We establish the teaching of developmental treatment of mathematical problems as an effective strategy for achieving higher thinking processes like the open-ended approach. And we identified the teacher's role in educational research using this research. The teacher is not only a recipient of research findings. Findings that teachers obtained as researchers guarantee their dissemination. Therefore, we need to ensure more situations in which teachers can exist as researchers. We examined effectiveness of activities in which students make up problems. However, the activity was not only for students. For both researcher and teacher, it was important to exert effort to find problems, to hypothesize on solutions, and to try to solve them.

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TEACHING WORD PROBLEMS IN THE FIRST GRADE: A CONFRONTATION OF EDUCATIONAL PRACTICE WITH RESULTS OF RECENT RESEARCH

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POSITIONS AND FUNCTION OF WORD PROBLEMS IN THE TEACHING OF MATHEMATICS

In the mid-seventies, a new curriculum for mathematics teaching was introduced into Belgian primary schools. This curriculum was strongly influenced by the ideas and concepts of the so-called new math. New instructional programs were developed and commercially distributed that reflected this new orientation. In a recent study, Janssens and Joillet (1983) analyzed a representative sample of six instructional programs frequently used in elementary school mathematics education in the Flemish speaking part of Belgium. The analysis concerned the teacher's manual and the children's text- and workbooks for the first grade, and was restricted to the teaching of word problems. In these six programs the impact of the new math is obvious from the outset: the children are immediately immersed in the study of sets and relations. After a certain time, more traditional topics of the subject matter of mathematics are taught, namely, numbers and operations. Numbers are introduced as characteristic of sets, and addition and subtraction as operations on sets. When the children are sufficiently skillful in writing and solving addition and subtraction number sentences with small numbers and have mastered reading to some degree, the first word problems show up in the instructional programs. The children are taught to solve verbal problems by searching, writing down, and computing the arithmetic operation "hidden" in the verbal text. As an aid, it is often recommended that the children make a schema of the problem in terms of an arrow diagram before writing down the appropriate number sentence. In other words, in the current mathematics programs for the first grade, word problems have an application function: it is expected that the pupils will learn to apply the acquired formal concepts and operations of formal arithmetic to cope intelligently with different kinds of problem situations by using the problem-solving procedure described above. At least two remarks can be made about this assumption.

First, we have doubts about the hypothesis that the formal concepts and techniques of arithmetic will transfer from solving traditional word problems to real-life problem situations. Several authors (Nesher, 1980; DeCorte & Verschaffel, 1984; Kintsch & Greeno, in press) have argued that being confronted with a traditional word problem during the mathematics lesson in the classroom setting differs considerably from the situation of a child facing a problem at home or on the playground. Therefore, transfer can hardly be expected. As a matter of fact, recent research has convincingly shown that many children at the end of the first grade still solve word problems without using the concepts and procedures of formal arithmetic taught in school; instead, they apply informal solution strategies that they discovered or invented apart from the curriculum (Carpenter & Moser, 1982; Riley, Greeno, & Heller, 1983; DeCorte & Verschaffel, 1982; Verschaffel, 1984).

Second, it is questionable whether it is proper to attribute to word problems only an application function in elementary mathematics. Indeed, several studies have produced evidence that young children who have not yet had instruction in formal arithmetic can solve simple addition and subtraction problems by means of informal procedures with manipulatives or verbal counting strategies (Carpenter & Moser, 1982; Riley et al., 1983; DeCorte & Verschaffel, 1982; Verschaffel, 1984). These findings suggest that word problems can, more than has been the case hitherto, be mobilized in the first grade to promote understanding of an to give deeper meaning to the formal arithmetical operations of addition and subtraction. As Carpenter and Moser (1982, p. 9) have stated, verbal problems "could represent a viable alternative for developing addition and subtraction concepts in school" (see also Greeno, 1978, pp. 24-25).

TYPES OF WORD PROBLEMS IN THE FIRST GRADE

The verbal problems from the six instructional programs were classified according to the types of simple addition and subtraction problems distinguished by Heller and Greeno (1978) (see also Green, 1978; Riley et al., 1983). The basic dimensions of Heller and Greeno's classification schema concern the distinction between three types of problems that differ in semantic structure: change, combine, and compare problems.

Change problems refer to situations in which some event changes the value of a quantity. For example: "Joe has 3 marbles; Tom gives him 5 more marbles; how many marbles does Joe have now?" In combine problems, two amounts are involved, which are considered either separately or in combination, as in the following case: "Joe has 3 marbles; Tom has 5 marbles; how many marbles do they have altogether?" Comapre problems involve two amounts that are compared and the difference between them, such as the following example: "Joe has 3 marbles; Tom has 5 more marbles than Joe; how many marbles does Tom have?"

Within each of the three problem types, further distinctions can be made in terms of the identity of the unknown, and, in change and compare problems, the direction of, respectively, the change and the difference. For example, the unknown set can be the start, the change, or the result set, and the direction of the change can be either an increase or a decrease. Based on these distinctions, fourteen different problem types can be distinguished (cf. Table 1).

Table 1

Types of Word Problems Distinguished by Heller and Greeno (1978)

Semantic Schema	Direction	Unknown	Problem Type
Change	Increase	Result set	Change 1
Change	Decrease	Result set	Change 2
Change	Increase	Change set	Change 3
Change	Decrease	Change set	Change 4
Change	Increase	Start set	Change 5
Change	Decrease	Start set	Change 6
Combine	-	Superset	Combine 1
Combine	-	Subset	Combine 2
Compare	More	Difference set	Compare 1
Compare	Less	Difference set	Compare 2
Compare	More	Compared set	Compare 3
Compare	Less	Compared set	Compare 4
Compare	More	Referent set	Compare 5
Compare	Less	Referent set	Compare 6

Our classification of the verbal problems from the six instructional programs in the categories of Table 1 revealed a remarkable one-sidedness. In most programs there was a substantial preponderance of change/result set unknown problems (e.g. "Pete had 6 apples; Ann gave Pete 2 more apples; how many apples does Pete have now?") and combine/combined set unknown problems (e.g. "Pete has 3 apples; Ann has 5 apples; how many apples do they have altogether?").

In several programs, very few or no problems of the following types occurred: change/start set unknown, change/change set unknown, and combine/part set unknown. In three out of the six instructional programs, not a single compare problem was found; in two programs there were very few compare problems, while only one program had a relatively good balance between the three semantic types. Since, in all six programs the verbal problems received much less attention than all the other aspects of the mathematics curriculum, our conclusion is straightforward: instructional practice seems to be characterized by a restricted, one-sided, and stereotyped supply of verbal problems. It is our conviction that not only is this situation undesirable, it also involves a risk in the sense that it promotes the development in young children of a restricted number of rather superficial solution strategies. By means of those strategies, the children are capable of solving quickly and without much thought specific of types of problems, namely, those that occur in the stereotyped supply. However, those strategies fail when pupils are given problems of a different, less familiar or more difficult kind. In this regard, the so-called key-word strategy comes immediately to mind: the child's selection of an arithmetic operation in this strategy is not based on a global semantic analysis of the problem situation but is guided by the occurrence of an isolated key word in the problem text with which an arithmetic operation is associated; for example, the words "more" and "altogether" are associated with addition, the words "less" and "lose" with subtraction (Nesher & Teubal, 1975; DeCorte & Verschaffel, 1982).

The analysis of the six instructional programs also showed that verbal problems are usually stated very briefly in children's workbooks. Consequently, the semantic relations between the given and the unknown quantities in the problem are often not made very explicit in the verbal text of the problem. As an illustration, let us consider the following problem: "Pete and Ann have 9 apples altogether; Pete has 3 apples; how many apples does Ann have?" In this problem text, it is not stated explicitly that Pete's three apples mentioned in the second sentence are among the nine apples that Pete and Ann have altogether. However, this problem can be reworded in such a way that its surface structure makes the semantic relations more obvious: "Pete and Ann have 9 apples altogether; 3 of these apples belong to Pete, and the rest belong to Ann; how many apples does Ann have?" (see also Lindvall & Ibarra, 1980). Our indepth longitudinal study as well as a recent investigation (DeCorte, Verschaffel, & DeWin, 1984) have shown that it is of utmost importance to state very explicitly the semantic relations between the given and the unknown components in the surface structure of the verbal text, especially for beginning first graders. The reason for this is that the semantic schemata (change, combine, and compare schemata) are not yet very well developed in the knowledge base of these inexperienced children, so they depend more on bottom-up to text-driven processing to construct an appropriate problem representation, while competent problem solvers process the verbal text largely in a top-down or conceptual driven way using their well-developed semantic schemata. Consequently, it can be recommended that writers of textbooks for the first grade pay more attention to the appropriate formulation of word problems and not concentrate only on the purely arithmetic aspects. Our recent paper cited above (DeCorte et all, 1984) contains suggestions concerning the direction in which one can search for rewordings that can help chidlren to construct an appropriate problem representation in a largely bottom-up way.

LEARNING TO SOLVE WORD PROBLEMS

In most of the programs that were analyzed, only very vague answers if any were given to questions like the following: What does a competent problem-solving process look like? By means of what teaching strategies can competent problem solving be promoted in children? What are the main difficulties to be faced in achieving competent problem solving in children, and how can they be overcome? In those cases in which some suggestions were given, they often were open to serious criticism. We will illustrate this statement for three different task instructions that are often recommended in instructional programs with respect to word problem solving, namely, direct modeling of the problem using physical objects, making a schema of the situation, and writing down a matching number sentence.

In one of the six programs, not only do the authors recommend the modeling strategy with blocks, but they also specify how to model them for different problem categories. We mention the authors' advice for three kinds of change problems. (1) "Pete had 8 marbles; then he lost 3 marbles; how many marbles did Pete have left?" According to the authors, problems of this type should be modeled as follows: first, the child constructs a group of eight blocks; then he removes three blocks, and finally he counts how many blocks there are left. (2) "Before the game, Pete had 9 marbles; after the game he had only 3 marbles left; how many marbles did Pete lose?" It is recommended that this problem be modeled in a totally different way: first, the pupil constructs a row of nine marbles and then a row of three marbles underneath it; the difference between the two sets is then determined by using a matching procedure. (3) "Pete has lost 4 marbles; now he has 3 marbles left; how many marbles did Pete have in the beginning?" This change problem, in which teh start set is unknown, should be solved by reversing the chronological sequence of events as described in the problem: first, the child creates a set of blocks that equals the number of marbles which Pete has left at the end, namely, three; then four blocks are added, and the child counts the total number of blocks. The modeling strategies recommended for the second and the third type of change problems do not at all correspond to the modeling procedure that kindergarten children and first graders apply spontaneously and often successfully to solve these problems (Carpenter & Moser, 1982; Verschaffel, 1984). In itself, this would not be too bad, if the authors would demonstrate that, by recommending the strategies mentioned, they are pursuing a specific goal. However, we have not been able to find a justification for what these authors do, namely, attempting to equip children with a number of totally different and very specific material solution strategies for different kinds of change problems.

In almost all the instructional programs, the children are taught at a given moment to solve verbal problems by making a visual representation of the relations between the quantities involved in the problem in terms of an arrow diagram. This raises to the following question: is it appropriate and justified to teach children one form of graphic representation, namely, the arrow diagram? In our opinion, the arrow diagram is very appropriate for representing the dynamic nature of change problems, but it is much less suitable for addition and subtraction problems with a different semantic structure, such as combine and compare problems. Other kinds of graphic representation are probably more appropriate for representing the main relations between the quantities in these categories of verbal problems, namely, the part-whole schema and the matching schema, respectively (DeCorte & Verschaffel, 1983-1984). When children are not given the opportunity to use their own forms of visual representation spontaneously or to discover the forms they find appropriate, but instead, have one generalized and uniform schema imposed on them, they are forced to reinterpret the verbal text of many of the problems, which is totally unnecessary for finding the solution. We grant that developers of instructional programs for mathematics teaching were probably unaware of this problem in the past. Indeed, until recently, educational practitioners as well as program developers were not acquainted with the finding of recent research that simple addition and subtraction word problems can differ significantly in terms of their underlying semantic structure.

An objective in all the analyzed instructional programs is that, at the end of the school year, first graders should be capable of solving simple addition and subtraction word problems by discovering, writing down, and computing the number sentence that is "hidden" in the verbal text. But, then, what number sentence matches the word problem? Indeed, one should take into account that number sentences can fulfill two different functions with respect to word problems: they can be used either as a formal, mathematical representation of the semantic relations between the quantities involved in the problem or as a mathematical notation of the arithmetic actions that should be or have been performed to find the solution of the problem. Sometimes one number sentence can fulfill both functions, as in the following example. A child is given the problem: "Pete had 12 pieces of candy; he gave 4 pieces to Ann; how many pieces did Pete have left?", and he solves it by decreasing twelve by four. In this case, the number sentence $12 - 4 = x$ represents the semantic structure of the problem as well as the arithmetic action performed by the chid. However, for many verbal problems, these two aspects have to be expressed by different number sentences. Consider the following problem: "Pete had some apples; then he gave 3 apples to Ann; now Pete has 5 apples left; how many apples did he have in the beginning?" The number sentence $x - 5 = 3$ represents the semantic structure of this change/start set unknown problem, but the arithmetic actions applied by most children to solve this problem match either the number sentences $5 + 3 = x$ or $3 + 5 = x$ (Vergnaud, 1982; DeCorte & Verschaffel, 1983). In none of the teachers' manuals accompanying the six instructional programs was any attention given to the relationship between number sentences and word problems. Here again, the program developers were probably unaware of the problem. It would certainly be useful to draw the teachers' attention to the fact that the relationship between number sentences and word problems is more complex than is usually assumed. At the same time, the teachers could be given some suggestions with respect to the following questions: It is desirable to teach the children the different functions of number sentences in connection with verbal problems? If the answer is positive, what is, then, the most appropriate teaching strategy? Should the teacher show a certain preference for one of the two functions of number sentences?

CONCLUSION

Our analysis of a representative sample of Flemish instructional programs for mathematics teaching in the first grade has revealed that simple word problems are treated in a rather off-hand fashion. Word problems are generally introduced only in the second half of the school year, after substantial teaching and practicing of the formal operations of addition and subtraction, including writing and computing number sentences. Word problems, then, are mainly assigned an application function: by performing such tasks children should learn to use their knowledge of formal arithmetic concepts and operations to solve real-life problems. We have criticized this conception in two different ways. First, we have serious doubts about the degree in which word problems fulfill this function appropriately in present-day instructional practice. Second, we think that the instructional programs seriously underestimate the potential role of verbal problems with respect to the acquisition of formal arithmetic concepts and operations in the beginning of the first grade. Indeed, it is our conviction that word problems, if taught appropriately, can contribute substantially to a better and deeper understanding of addition and subtraction in children.

Another shortcoming of current instructional programs relates to the restricted, one-sided, and especially the stereotyped supply of word problems. A very important disadvantage of this situation certainly is that it facilitates the development of inappropriate and superficial solution strategies.

Finally, we have found that the manuals for the teacher, which are part of the instructional program, generally contain few directions, aids, and hints for guiding the teaching-learning process. And, when guidelines for the teacher are included, they are very often open to criticism. This certainly has to do with the fact that, in developing their instructional programs, the authors could not yet take into account the important findings of recent research in the instructional psychology of children's problem solving with respect to addition and subtraction word problems. Therefore, it is at least desirable that developers of instructional programs and also elementary school teachers become acquainted with the well-documented research results that have been produced and take them into account in the development or the revision of instructional materials and in the planning of instruction on word problems.

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RESEARCH LEADING TO NOVEL CLASSROOM AND INSERVICE ACTIVITIES

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This paper describes some segments of a large comprehensive project, which is responsible for the mathematics instruction in most of the 7th, 8th, and 9th grade classes is Israel. The rationale guiding the project is that development, implementation, evaluation, feedback, and research take place in interlocking and ongoing cycles. These cycles aim to improve "conditions", "means", processes and products of learning mathematics in the relevant population. Research is planned to effect, and to be used in, the development and implementation stages. Adopting the view that "any changes in curriculum and instruction must be through the minds, motives, and activities of teachers" (Shulman, 1979), a major part of our implementation and research activities are directed at the teacher.

> "The teacher today is faced with a curriculum content richer than ever before, with a variety of possible teaching strategies greater than ever before, with a population more heterogeneous than ever before and on top of all this, an alarming rate of change. It is, therefore, clear that even if the initial training were of the highest standards, there might be some need for 'topping up' with inservice training and other guidance and tutoring activities" (Bruckheimer & Herschkowitz, 1983).

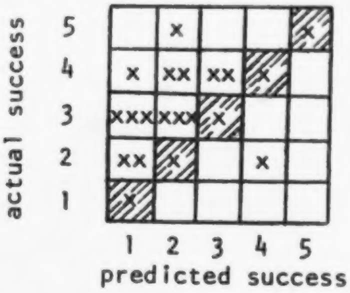
We need in-house research and evaluation in order to guide us in the choice and priority of our activities. This implies ongoing evaluation and long-term research strategies. Two examples of such research strategies, which are different but yet affect each other and together affect project activities, will be described.

Example 1: A Model Relating Teacher Prediction and Student Difficulties. Studies within this model have been conducted since 1978 (Zehavi & Brickheimer, 1981). The basic premise is that some of the concepts and parameters used in evaluation are essentially not objective, but subject to the interaction of student, teacher, and program. We start by asking the teachers to predict the success rate of their students, on each item in a test related to the curriculum, and to discuss the expected cognitive difficulties. Then the test is applied to the classes. The data is organized graphically to illustrate two types of analysis:

(a) analysis of one class over all the items;

(b) analysis of one item over a group of classes.

Type (a) information is fed back to the teacher by his teacher-tutor, who observes the class. They discuss the results, look for explanations in the case of those items which were over- or underestimated, and examine their implication in the teacher's own classroom.



Note: A five-point scale is used, from 1 (less than 20% answer correctly) to 5 (more than 80% answer correctly)

Figure 1. Graphical Representation of Data for One Class Over 20 Items

As an example, consider the graphical representation of data for one class over 20 items (Figure 1). In this class, for which the graph depicted severe underestimation, we found a "good" class and a "good" teacher. The class did very well on the test and the teacher was experienced. But he was sure that the best way to teach was to explain and explain again. He did not believe that his students were capable of doing anything without him first explaining every detail. When he became aware of the situation as represented in the graph, he realized that he could and should provide his students with more challenge reflecting their ability.

A teacher who has a "investigative spirit" can use the method by himself. The method itself also encourages the teacher to become investigative. If we concentrate on one item over a number of classes and find a mismatch, this can become the start of a more extensive investigation for possible causes. In fact, such a situation led to follow-up studies. The same method was used to examine a whole topic for which some items indicated a fairly consistent mismatch. In general, such topics were related to "modern" mathematics and "new" instructional methods. A further application of the basic method was designed to verify the suggestion that there is consistent overestimation or underestimation of items by teachers on newer topics in the curriculum (Zehavi & Brickheimer, 1983a). The junior high algebra curriculum includes both "modern" and "traditional" topics. Thus, the first chapter in ninth grade algebra is "modern" and deals with general function concepts. We decided to apply the method, in order to obtain a comparison between this chapter and one from the same course that can be loosely described as "traditional". One subtest on each topic was prepared. We can see the difference between the two topics in Table 1.

Table 1

Comparison of Profiles of Two Topics Based on Expected-Actual Data

	Topic 1 "new"		Topic 2 "traditional"	
difficulty / familiarity	easy	hard	easy	hard
familiar	over-estimation	over-estimation	precise estimation	precise estimation
partially familiar	under-estimation	over-estimation	precise estimation	under or overestimation
unfamiliar	under-estimation	under-estimation	under of overestimation	under or overestimation

The situation for topic 2 was as may be expected. The three easy familiar items were estimated precisely by the teachers and the other three were over-estimated or underestimated almost equally. It is not unreasonable that, for unfamiliar questions on a familiar topic, some teachers will overestimate the outcome and others will underestimate, reflecting individual teaching methods and personality. The results for topic 1, on the other hand, display a mismatch: the familiar was seriously overestimated and the reverse was true for the unfamiliar. Although the topic was featured in the textbook for more than ten years, teachers and (hence) students still feel uncomfortable and are "out of touch" with each other and with the program. This indicates that, in such topics, teachers needed still greater help in order to achieve a better understanding of student difficulties. This procedure identifies priority areas of inservice activities and helps to determine training strategies.

For several items, serious inconsistent discrepancies were found between teacher expectation and student performance. As a result, another study was carried out to see if teacher view of student difficulties had some connection with teacher education and experience (Zehavi & Bruckheimer, 1983b). The students were given the questionnaire, Functions: calculations and substitutions consisting of six items. The teachers were asked to consider the following three of the six items.

(1) Given that $f(x) = ax^2 - 3$ and $f(2) = 29$, find $f(6)$.

(2) Given that $f(x) = ax^2 - 15$ and $f(3) + f(4) = 120$, find a.

(3) Given that $f(x) = ax^2 + bx$, $f(4) = 8$ and $f(1) = -7$, find a and b.

For each item, the teachers were asked to estimate the percentage of student success. They were also asked to justify their expectation by estimation of possible difficulties in the following four categories: unable to get started, use of irrelevant procedure, mistakes in the use of function notation and technical algebraic mistakes.

Actual student success percentages on the three items and teacher expectation are given in Table 2. The teachers slightly overestimated student achievement in using the knowledge they had been taught. On the other hand, student-teachers lacking experience, completely underestimated student ability to even start working on such problems. Each of the two groups of teachers and student-teachers was divided into two subgroups according to their education - university degree in matheamtics or college certificate. A comparison of actual student difficulties with the teacher view of these difficulties provided an explanation for discrepancies in teacher education based on their experience and education. The significance of this study is that it can help teachers not only to be aware of student cognitive difficulties, but also to be aware of issues where their own conception of those difficulties does not correspond to reality. A further application of the model as a tool for inservice activities can be as follows:

Table 2

Student Performance Versus Teacher and

Student-Teacher Expectation

	Item 1	Item 2	Item 3
students (n=225)	64%	43%	38%
teachers (n=25)	72%	54%	55%
student-teachers (n=30)	20%	23%	14%

For items that the previous application of the model indicated some "trouble" in estimation, tests are constructed and are given to a sample of classes. Teachers are invited to workshops just before they are going to teach the specific topic in their classes. In the workshop, they first discuss the difficulties they expect students to have and estimate percentages of success. These are compared with the actual findings with students. Figure 2 presents the findings for a test on common algebraic techniques for grade 8. Student performance is then analyzed in light of the teacher expectation.

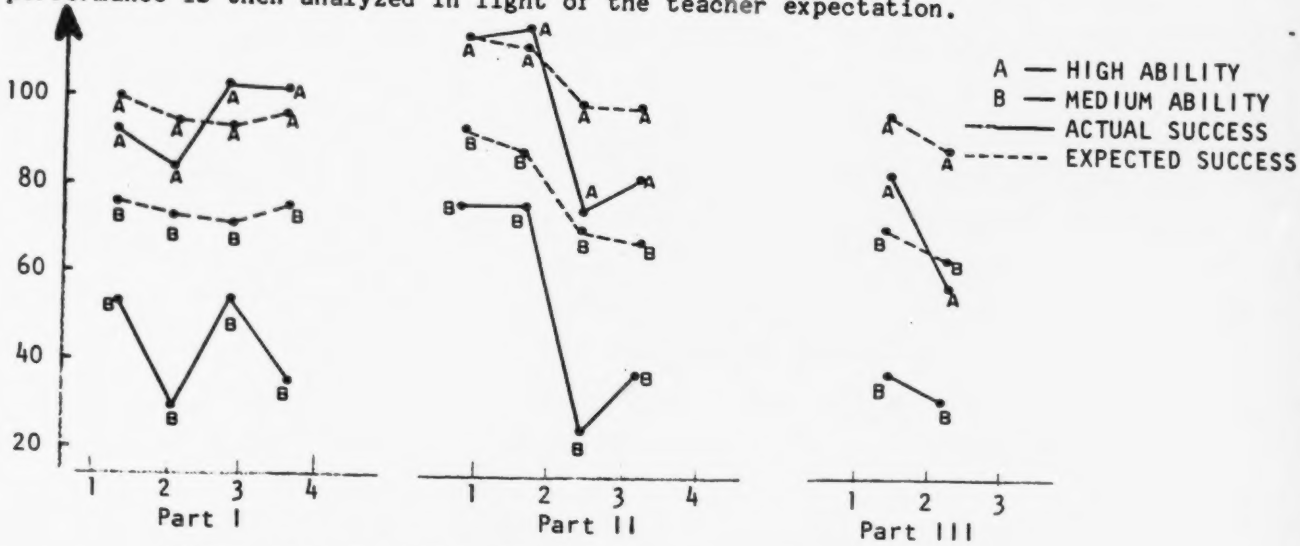


Figure 2. Student Performance and Teacher Expectation

It is expected that, in such workshops, teachers will become more aware of student difficulties and consequently, will improve their teaching strategies and that this will eventually lead to better teaching/learning processes. To summarize, the model described can be used iteratively to identify needs and methods to overcome them in the ongoing development of the project.

Example 2. Inservice Guidance: The Consumer View (Ben Chaim, Hershkowitz, & Bruckheimer, 1983). One of the two major battle cries of the education reform in Israel was the "academisation of junior high school education"; that is, the provision of academically trained specialist teachers from the beginning of grade 7, accompanied by an appropriate curriculum. In fact, after some fifteen years, there are still not enough qualified teachers (40% have a university degree in mathematics, 25% "graduated" from teachers' college with mathematics as a specialization, 35% "graduated" from an elementary teachers' college). More than 85% teach only mathematics (Hershkowitz & Israeli, 1981).

We thus have a wide spectrum of inservice needs, from extremely "ill-equipped" teachers, to those whose requirements are quite sophisticated. We try to face these needs with a wide range of inservice activities which include an inschool guidance system, written materials (periodicals and special books for teachers), and regular inservice courses and workshops.

The main goal of this study was to examine the impact over time of our ongoing guidance system in order to improve it. In addition, the research tool which was to be developed, should serve as a base for the continuous development of other research tools for this type of evaluation. We chose to obtain the information directly from the main consumer of the inservice guidance system, the teacher. From a larger sample, 69 teachers (representing a population of about 1000) were selected, in order to create three groups

- Group A, without inschool guidance,
- Group B, with at most two years of guidance,
- Group C, with more than two years of guidance.

The teachers in each group were matched according to the following background variables: level of education (university, teachers' college with math, elementary teachers' college), experience (1-4 years, 5-9 years, more than 10 years), type of school (i.e. percentage of socially deprived: less than 35%, between 35% and 70%, more than 70%). This means that for each teacher in group A with a given level of math education, experience and school type, there was a teacher with the same data in group B and in group C.

The instrument used in the study was a questionnaire consisting of two parts:

- background information necessary for the above matching.
- the main part of the questionnaire containing 23 items. Each item was a description of an activity to which the guidance system might possible contribute, and the teachers were asked to scale the items according to importance (not important, important, and very important).

The teachers in groups B and C were also asked to scale the items according to the role of the guidance system (no contribution, contribution, considerable contribution). Examples of items are:

- planning subject matter for the coming term (trimester),
- increasing teacher awareness of specific student learning difficulties,
- assistance with the integration of mathematical games with other teaching

strategies.

The 23 items were grouped to form 5 main guidance "function" categories:

1) Planning - teacher guidance in planning his teaching (trimester, single lessons, tests, worksheets).
2) Updating - the provision of relevant information (the primary - junior high - senior high curriculum interface, the curriculum in other schools, developments in the project).
3) New teaching strategies - individualized learning, group learning, special strategies for different ability groups/classes.
4) Implementation - of new supplementary and/or enrichment material created by the project team.
5) Types of guidance - various guidance activities in school; i.e. individual guidance, demonstration lessons followed by discussion, etc.

It was assumed that teacher opinion of the importance of guidance system functions reflects the "desired state", or his view of an objective need. His assessment of the contribution of the existing guidance functions refelcts the "real state". The relative differences between the two, for each guidance function, gives some indication of teacher need. For each teacher in the sample, a mean "importance" and "contribution" grade for each of the above five functions was calculated. (A 1-3 scale was used in all calculations; e.g. not important = 1, important = 2, very important = 3). The main findings are summarized as follows.

First, both the "unguided teachers (group A) and the "guided" teachers (groups B and C) saw the five guidance functions as more than "important", with the guided teachers attaching a little more importance, but not significantly so.

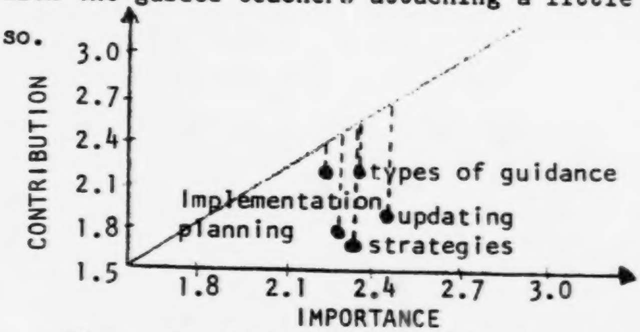


Figure 3. Contribution - Importance Comparison for Guidance Functions

Second, a comparison between "importance" and "contribution" of the different functions for the group of guided teachers (N=46) is shown in Figure 3. The absolute value of this difference for each function has little meaning, but there is some significance to these differences relative to each other. Thus, there is a clearly greater need to find ways of contributing to the planning, updating, and strategy functions than to the other two. Third, a comparison between the needs of teachers who had received at most two years guidance (group B), and teachers who had had more than two years of guidance (group C), is shown in Figure 4.

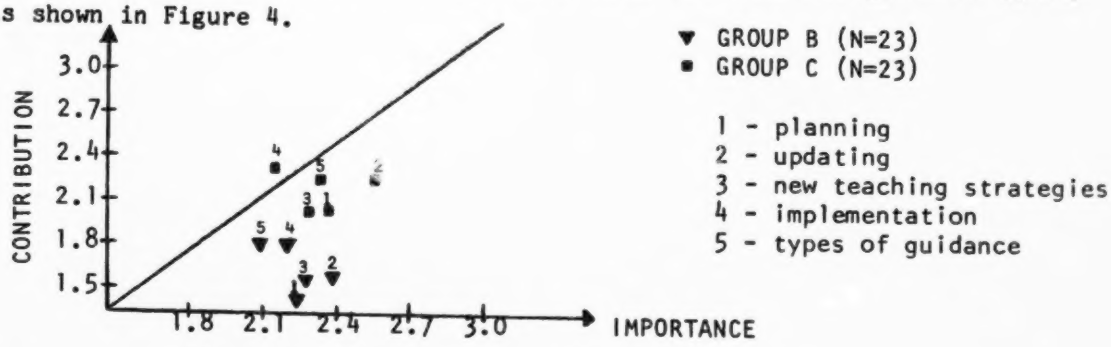


Figure 4. Teachers' Need for Group B and C

It is clear that group C teachers saw the guidance functions as more important, and the guidance system as contributing more, than did group B teachers. Even more significant is the fact that "need", as described above, was considerably greater for group B than for group C, indicating that the guidance system had a cumulative effect. This does not necessarily imply that the contribution of the guidance will continue to increase with the years, or that the teacher "need" will continue to decrease. Assuming a relatively static teacher population and static curriculum, we might well reach saturation, with little or no further change in contribution or need. But the teacher population is far from static, and the curriculum project philosophy is based on gradual evolution and renewal.

The conclusions from this study were in two directions. First, changing the project's inservice activities in a way that "guidance functions", for which the teachers express a greater need, will be given more emphasis than other functions. (In a situation of shortage in tutoring manpower and resources, we have to play our efforts very carefully.) For example, a new series of teacher texts, Mathematics and Test, was developed. These texts guide and help the teacher in planning and producing a suitable test for his class, on each topic of the curriculum. This development was followed by implementation activities in workshops on this and related topics, and by research activities, of which parts were described in the first example in this paper (Buhadana

& Zehavi, 1984).

The research tool (the questionnaire) of this study served as the basis of a research tool in an additional study. The new study, which examined the effect of some of the summer inservice courses, consisted of one part which was developed from the above mentioned questionnaire and other parts which examined teacher knowledge and confidence in the topics to be taught (Fresko & Ben-Chaim, 1984).

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MATHEMATICS GAMES: FROM THE CLASSROOM TO THE LABORATORY AND BACK

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In 1974, a multi-research project on the use of games to teach mathematics was initiated (Bright, Harvey, & Wheeler, 1983). Primarily, this project began in order to investigate what mathematics was being learned from the kinds of games teachers were then using in their classrooms. Observations of teachers and reviews of published games suggested that the mathematics taught by those games was almost always knowledge-level content (Bloom, 1956) and that the primary use of games was for drill and practice.

In large part, these kinds of games and the uses of them may have been dictated by the lack of a substantive body of research focusing on the effectiveness of games in teaching mathematics. Teachers, in conversations, suggested that they perceived that games could only be used for drill and practice of skills. One principal goal of the research project, then, was to determine if the range of mathematics known to be learnable through games could be expended beyond knowledge-level content embedded in drill-and-practice situations. When the project began, there were essentially no microcomputers in schools, so an implicit decision was made to restrict attention to non-computer games. Extension of conclusions to computer formats is yet to be done.

In order to understand the organization and results of the research, several definitions need to be made. A game is defined by the following seven characteristics:

1. A game is freely engaged in.
2. A game is a challenge against a task or an opponent.
3. A game is governed by a definite set of rules.
4. psychologically, a game is an arbitrary situation clearly separate from real-life activity.
5. Socially, the events of a game situation are considered in and of themselves to be of minimal importance.
6. A game has a finite state-space (Nilsson, 1971); the exact states reached during play of the game are not known in advanced.
7. A game must always end in a finite number of moves.

An instructional game is a game for which the instructional objectives have been determined; these instructional objectives may be cognitive or affective ones and are determined by the person(s) planning instruction, before the game is played by the students who receive instruction from the game. Whether the game helps students attain those instructional objectives is an empirical question, but it is important that the instructional planner(s) identify these objectives prior to using the game.

The cognitive or taxonomic level of content is defined by Bloom's taxonomy (1956). The six levels are knowledge (e.g., recall of facts), comprehension (e.g., use of an algorithm), application (e.g., choosing the most appropriate algorithm from among several possible algorithms), analysis (e.g., recognizing unstated assumptions), synthesis (e.g., creating a simple proof), and evaluation (e.g., choosing the most "elegant" of two correct proofs). The cognitive level of a game is the highest cognitive use of the content that a player would need in order to play the game efficiently and well. The instructional level of a game can have one of three values. A game is used at the post-instructional level if it is used after the primary instruction designed to produce mastery of the material for the students, at the co-instructional level if it is used along with that instruction, and at the pre-instructional level if it is used prior to that instruction. To determine the instructional level of a game, it is necessary to know the backgrounds of the students being taught; the instructional level of a game applies to a group of students and is dependent on the instruction that has been provided.

One thrust of the research conducted from 1979 to 1983 was to examine games which were categorized as being in one of the combinations of the lower four cognitive levels and the three instructional levels. (The combination of analysis and pre-instructional levels, however, was not studied.)

During the research, it was necessary to develop a procedure for identifying or constructing a game which teaches content at a given cognitive level. This process may be of considerable use to researchers and teachers who want to design games to fit a particular instructional objective. The first step is to identiyf situations that would, or would not, reflect the use of the content at the given cognitive level. For example, at the application level, when the content is converting among common fractions, decimal fractions, and percentages, an appropriate situation would be to have students select equivalent numbers from among a list of numbers in the three forms. An inappropriate situation would be to ask for a number in one form to be changed into a number of a specified alternate form; this would be comprehension-level behavior. The second step is to design a game setting in which the appropriate situations will be encountered repreatedly and are useful in playing the game efficiently or well. A danger which should be avoided is having the repeated situations become so familiar that the cognitive level is reduced to simple recall. Once the game is developed, it must be reanalyzed to assure that the appropriate cognitive level is required for efficient or good play.

Implementation of this process depends on indepth familiarity with Bloom's definitions of taxonomic levels. It is equally important to have a thorough knowledge of the instructional objective and of the backgrounds of the students. Knowledge of the kinds of instruction that have been given is essential for identifying behaviors at the appropriate cognitive level, since what is recall for one group of students may be problem solving for another.

As an example of this process, consider the game AVERAGE HANDS, presented after the reference list. In this game, players must interpret the numbers on teh cards in their hands within the context of the goal task and then extrapolate as they predict or estimate averages within specified ranges. Interpretation and extrapolation are comprehension level tasks, so this game is at the comprehension level. Trying to play the game at a lower level (i.e., knowledge) would be inadequate, since typical mathematics instruction does not deal directly with this task. The difficulty of the game can be varied by using different sets of cards and different scoring rules.

The process of selecting games for the various parts of the research built on a variety of completed studies on instructional games, and it allowed a systematic expansion of those studies. The total collection of studies completed to date numbers several dozen. From this collection, a variety of conclusions about appropriate uses of games can be drawn. Direct extension of conclusions to computer games seems likely, but it is certainly not guaranteed.

It should be pointed out that the research involved repreated use of games in a classroom. This is an important consideration for implementation of the games by other teachers. Single use of a game might not be effective. Too, the learning produced by the games was measured either by differences in pretest and posttest performance or by comparison of performance of games and non-games groups. Performance of individuals was not the focus of the research. Further, some of the games were at the higher cognitive levels. This may have important implications for the teaching of problem solving.

A possibly surprising finding is that several characteristics of the games or the game situations do not seem to alter the effectiveness of games. First, and perhaps most important, is that opponents can be randomly chosen (e.g., Bright, Harvey, & Wheeler, 1980a); that is, the level of previous success of players with the content does not seem to be critical for most games to be effective teaching devices. This is an important result because it gives the classroom teacher considerable flexibility in using the games. Second, the amount of verbalization among players generated by the game does not seem to be critical (e.g., Bright, Harvey, & Wheeler, 1980b). This means the teacher does not have to monitor the verbalizations and does not have to try artificially to increase the amount of verbalization. This means the teacher does not have to be closely involved in the play of the game. Third, the format of the game does not seem to be critical to the effectiveness of the game (e.g., Bright, Harvey, & Wheeler, 1982). The cognitive level at which the players must use the content may be important, and the context of the game (e.g., fantasy) may interact with the gender of the players, but the format of the game does not seem to be critical.

There are also several research results that do seem important in determining how to use games effectively. First, games can be used to teach content at a variety of cognitive levels (e.g., Bright, Harvey, & Wheeler, 1982). However, a game must demand that the players use the content at the appropriate level. It is important, then, to analyze the cognitive level of a game and not to expect it to teach beyond that level. Second, many games may be more effective if external support (cf. fraction bars) is provided which will help players dealw ith the content of the game (e.g., Bright, Harvey, & Wheeler, 1981). This result is complicated by the fact that the use of external supports might be differentially effective for different subgroups of players; for example, high achievers versus low achievers. These kinds of interactions cannot be logically determined ahead of time; only research investigations are likely to reveal them. Third, a game should be repeatedly useable. If it can be used only once, students are not likely to become involved with the content; rather, they may focus more on understanding the rules. The studies completed to date have all involved repeated use of games; as the cognitive level of a game increases, effects tend not to be observed unless repeated playing sessions are used. Fourth, games at higher cognitive levels can be used along with regular instruction at lower cognitive levels to teach the higher level material that might otherwise not be learned (cf. Bright, Harvey, & Wheeler, in press). This finding may be especially importnat in assisting the development of problem-solving skills, since higher cognitive level skills are likely important to successful problem solving.

Taken as a whole, the research clearly shows (Bright, Harvey, & Wheeler, 1982, 1983, in press) both that higher-level (i.e., application- and analysis-level) material can be learned from games and that games can indeed be effective as a primary teaching technique and as a readiness activity for some mathematics content. The next step in translating these results into classroom practice is to make the games widely available. Of course, teachers might read published research reports and identify the games used. Teachers might also find out about the games at professional meetings or through reading articles published in professional yearbooks or journals. Certainly some teachers have been reached through these means, but the numbers seem to be small.

An additional route availble is to prepare versions of the games for consideration by teachers along with other supplementary instructional materials. Games can be added to any "in-place" curriculum and can be used quite flexibly. A game which promotes learning will be useful whenever the instructional objective of the game matches the instructional objective of the teacher, regardless of whether that objectiv deals with basic skills or problem solving. The supplementary route, then, seems to one most likely to succeed.

At the same time, probably more teachers, at least in the United States, examine and use commercial supplementary materials than materials produced by professional organizations. In order to distribute the games as widely as possible, therefore, a decision was made to supplement the dissemination of the research findings with concurrent commercial publication of various versions of the effective games. Fortunately, a publisher has accepted this idea; this reaffirms the research-supported high quality of the games developed. The preparation process for publication puts constraints on the particular forms of the games that were not present in the research studies. In particular, the design of the games and the amount of equipment needed to play them msut be kept to a minimum; otherwise, the produce becomes too expensive to be attractive to the publisher or to potential users. These concerns were of minor interest during the research since only a few copies of each game had to be prepared, and there was equipment at the research sites that could be used repeatedly. For example, one of the devices that was used in many of the games was a clear plastic spinner developed for Developing Mathematical Processes (Romberg, Harvey, Moser, & Montgomery, 1974, 1975, 1976), an elementary mathematics program developed with support from the U.S. federal government. These spinners are moderately expensive, however, so adaptation of some of the game formats to use less expensive equipment was necessary. At the same time, the research-supported desirable characteristics of the games need to be retained so that teachers can be assured that the games are effective.

The most obvious alteration in the format of the games as they are prepared for publication is to move from spinners as generators of random information to grids from which information is selected by rolling dice. As noted earlier, the physical format of a game does not seem to be an important determinant of its effectiveness. Hence, this alteration does not seem to be a critical one. A second difference is the adaptation of the effective games to other content which is at a similar cognitive level. For example, one very effective game type is the TIG games, modeled on Broadbent (1972). The apparent universlity of the effectiveness of the TIG games (Bright, Harvey, & Wheeler, 1979, 1980c, in press) generally supports the use of this format for a variety of content. Consequently, it is being adapted to a wider range of fraction content than was studied directly in the research. This kind of adaptation seems quite appropriate and well supported by the research evidence.

The most important research process to be transferred to the publication of the games, however, seems to be that of developing a game to fit a given cognitive level and of analyzing the cognitive level of a particular game. The games prepared for publication are each organized around clearly identified instructional objectives and are developed so as to teach those objectives. It is recognized, however, that teachers might have broader instructional objectives, or might want to provide more instruction than had been considered in the research. Hence, more variations of each published game are necessary.

In conclusion, it seems that both the process of developing a coherent research rpoject and that the particular results arising out of that research have been important in translating the research into classroom practice.

Neither one of these alone would have supported adequately the process of translation. hence, while it is useful for teachers to knew the results of research, it is equally useful for them to be immersed in at least some of the detail of that research. Only by understanding the whole context of the research can translation into practice be truly effective. In particular, in the games project, it became important to identify clear and attainable instructional objectives for each game, to determine the cognitive level of each game, and to create games which would be interesting for students to play repeatedly. The effort devoted to each of these details probably has been an important element in the ultimate effectiveness of the instruction that ensued. Too, the decision to have students play the games repeatedly, and the experimental verification that this instructional procedure results in learning of the instructional objectives, allow a clear translation of the games into classroom practice. The translation is, of course, not completely teacher-independent, but enough specificity can be given to teachers to ensure that they can use the resulting games effectively.

The research project and the translation of its results into practice perhaps will give researchers and teachers a model for developing instructional strategies that are effective. If so, the work will have been more successful than even originally intended. We hope at the least, however, that teachers will use the games to help students learn and enjoy mathematics.

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Average Hands

(2 or 3 players)

You will need

digit cards
paper and pencil
score sheet

Game rules

1. Shuffle the cards. Deal twelve cards to each player.

2. Each player looks at his or her cards and arranges them into three hands: one hand of three cards, one hand of four cards, and one hand of five cards.

3. Each player's three hands are laid face down in front of that player.

4. Then scoring begins for that round. Continue playing rounds until one player has 100 points. That player is the winner. Use the scoring directions at the right.

How to score

a. Each player turns over his or her three-card hand and finds the average of the three cards. The players round their averages to the nearest whole number (round $3\frac{2}{3}$ to 4).

b. The player with the lowest average receives 3 points. If there is a tie, each player in the tie receives 3 points.

c. Scoring steps a and b are repeated for the four- and five-card hands with these exceptions: The player with the lowest average for the four-card hand receives 4 points. The player with the lowest average for the five-card hand receives 5 points.

from Romberg, Harvey, Moser, & Montgomery (1974, 1975, 1976)

EPILOGUE

AN OPEN DISCUSSION

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The final session of the Using Research Group of the Professional Life of TEachers Theme at ICME-5 was devoted to an open discussion including all participants. The group addressed itself to four questions: (1) Do teachers use research?; (2) What do teachers want from research?; (3) Should the products of research be given more emphasis in helping teachers?; and (4) Should teachers be partners in research? This section is not a record of the minutes of that final meeting, but rather a discussion that hopes to capture the spirit of that session.

DO TEACHERS USE RESEARCH?

There is little question that the research process in mathematics education has not reached its full potential, maturity, and applicability to the improvement of the teaching and learning of mathematics. There are many reasons for this. First, a great deal of research in mathematics education is of the "one-shot" research exercise-type because it is part of a graduate degree program; second, some research by its very nature is designed to uncover theoretical principles that help in the understanding of learning but does not have immediate applicability to the classroom; and third, even if research may be applicability to classroom problems, the dissemination-utilization phase of the research process has not sufficiently evolved to insure an immediate availability of research to those who may employ it in the mathematics classroom.

On the other hand, there are some promising world-wide trends that demonstrate that research in matheamtics education does have applicability and its utilization is changing classroom practices. Some evidence of this change is as follows.

(1) The informal survey by Williams (this monograph) revealed that large percentages of teachers (90 percent or more) in Australia, Scotland, and the United States felt that research results had been absorbed in their teaching styles and had presumably effected changes in mathematics education.

(2) The Assessment of Performance Unit (APU) of the Department of Education and Science in England which was established in 1975 to promote the assessment

of achievement of children has resulted in products that are being used in the classroom (see Clegg, this monograph). Videotapes of useful assessment techniques have been prepared for teachers. The techniques themselves are being used in many classrooms including the schools of Adelaide.

(3) Several reports at this meeting gave evidence that research conducted in Belgium, Israel, Japan, United States, and Canada related to children's strategies in learning addition and subtraction, or rational numbers is reading a consensus among researchers in ways that are proving useful to teachers in working with children. Furthermore, research related to evaluation, assessment, testing, classroom games for learning are finding their paths into classrooms in many useful ways.

WHAT DO TEACHERS WANT FROM RESEARCH?

There appear to be three points of view in regard to making research available to teachers and other school practitioners: (1) there are those who believe that the raw ideas, the ethos, the constructs of research as soon as available should be presented to teachers who may employ them in the classroom and reach their own conclusions as to their usefulness and applicability; (2) there are those who advocate a more conservative view that research should stay strictly in the province of researchers until such time as it has matured to produce stable, definitive results and that at that time and only that time should be made known to teachers; and (3) there are those who feel that (1) and (2) are inappropriate because the results of research should be products (textbooks, instructional materials, tests, guides, etc.) and that these products should be produced for the teacher's use.

What teachers want from research is further complicated by the type of research that is performed. This often dictates what may prove useful to teachers. For example, there is resaerch that is purely decision oriented. A very specific, well-defined problem is posed, and research is designed to solve that problem. Much of the research performed by APU in England is of tha kind (see Clegg, this monograph). Second, there is problem identification research in which the emphasis is placed upon studying a problem and the conceptual frameworks giving rise to that problem. The researcher may not be the appropriate person to design a remedy for the problem that has been studied. In fact, it may often be the practitioner who is in the best position to provide a remedy. Third, often research provides a new frame of reference for viewing mathematics and the ways children think about mathematics. However, this research has limited meanings to teachers until they have developed an understanding of the new frame of reference. This requires a re-education of teachers to accept the pedagogical implications of the research.

SHOULD THE PRODUCTS OF RESEARCH BE GIVEN GREATER PRIORITY?

Ideally, research would lead to a product that embodies the results of that research, but the number of cases where this actually occurs is small. The development of instructional games in the United States as a result of prior research is an example of research that was clearly directed toward a product. The group discussed this development and whether or not the game research model could be applied to the development of other products, such as textbooks. The general opinion was that the game model was too limited to be useful for the development of larger segments of the curriculum. It did seem abundantly clear, however, that the development of products should be one of the targets of research, if research in mathematics education is to have a significant impact on learning in the mathematics classroom.

SHOULD TEACHERS BE PARTNERS IN RESEARCH?

The desirability and the soundness of involving teachers in the research process was uncontested by the group. The basic problem revolves about "how" and "in what manner?" this involvement can take place. In the discussion below, some of the salient points that the group identified will be given.

(1) Teachers are busy people, often teaching 25-30 hours per week in most countries of the world. To expect that they can squeeze from their busy schedules time for research is not very realistic. In addition, most school administrators are not convinced of the value of research to improving instruction so the problem of winning their support further complicates the question.

(2) Many teachers would like to participate in research, assuming that they were provided sufficient time to participate. This participation could take place in a variety of ways:

(a) Teachers can do research projects of their own, particularly if part of advanced degree programs at colleges or universities.

(b) Teachers can be members of steering groups for research projects, therefore, providing the perspectives of the practitioner.

(c) Teachers should be part of any research effort that will involve the development of materials for students to use in the classroom.

(3) Initially, the role of the teacher in research may be limited by practical considerations of time and funds. However, if more funded research programs were formulated in terms of participation, then it would be more likely that educational leaders would see the benefits of teacher participation. With such innovations as the career-ladder movement in the United States, the teacher-researcher could become a more viable position of responsibility for teachers.

CONCLUSION

A Research-Dissemination-Utilization Model is evolving in mathematics education. The tendency for professionals to participate in one phase of the model to the exclusion of the others makes its development cumbersome. But it seems that its development is necessary if research is to make a signifcant impact upon the improvement of instruction. It is hoped that the Using Research Group of the Professional Life of Teachers theme at ICME-5 has provided a small stimulus toward the solution of that problem.